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# Aircraft Flight Test Trajectory Control

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SECTION 1  
INTRODUCTION

1.1 BACKGROUND

Flight Test Trajectory Control (FTTC) is an emerging, pilot-aiding technology valuable for improving flight test results and applicable to maneuver control and guidance in a number of contexts. This technique has provided the means for flying maneuvers consistently, precisely, and repeatably from flight to flight. Two versions of these controllers have been used: a closed-loop automatic system and an open-loop system providing manual piloting information. A closed-loop system used to collect performance, pressures, and loads data from the highly maneuverable aircraft technology (HiMAT) vehicle is described in [1]. The application of the open-loop system on the NASA F-111 transonic aircraft technology (TACT), F-15 airframe/propulsion system interaction studies, and F-15 shuttle tiles test programs are given in [2].

Originally, the open-loop flight-test-trajectory guidance algorithms were developed on-line, in a piloted simulation using cut-and-try techniques that were not only man power intensive, but often produced less than optimal controllers. A closed-loop system designed using one-loop-at-a-time classical design approach is documented in [3]. Full-state feedback approach for closed-loop system design using linear quadratic synthesis is described in [4]. Both these approaches have limitations in terms of design methodology and controller complexity.

The work reported in reference [2] and more recent ongoing work have included detailed modeling of all the piloted aircraft subsystems for suitable control law development. This research has identified the maneuver modeling as a significant technology needing further clarification and development. The maneuver modeling aspect of trajectory guidance and control law design will be emphasized here in describing this developing technology.

In the present work, closed loop mechanization is carried out with the pilot in a supervisory role. The thrust here is in four areas

1. Maneuver modeling,
2. Application of modern linear multivariable synthesis,
3. Techniques for the development of reference command and gain-scheduled perturbation controllers, and
4. Exploratory investigation of the emerging nonlinear system design techniques via prelinearizing transforms.

## 1.2 SUMMARY OF RESEARCH ACCOMPLISHED

Maneuver modeling for all the given flight test trajectories were successfully carried out. In the present work 64 straight and level trims, 51 level turn trims at a load factor of 2, and 30 level turn trims at a load factor of 4 were employed. A 3-D linear interpolation was employed. Thus a flight condition is characterized by altitude, Mach number and load factor. Linearized aerodynamic coefficients were also stored. This data in conjunction with kinematic and some dynamic equations are then used to generate state and control histories for all the flight test maneuvers.

Two multivariable synthesis techniques were used to obtain output feedback linear perturbation controllers viz.

- 1) Constrained eigenstructure assignment (Shapiro & Andry, 1982)
- 2) Minimum error excitation technique (Kosut, 1970)

Out of these two, the constrained eigenstructure assignment technique was found to be the hardest one to iterate on primarily due to the current lack of understanding of the explicit relationship between eigenvectors and the desired time response. This difficulty, while not crucial in full state feedback design can become an extremely important element in output feedback

design. Due to its high sensitivity to the input eigenvectors, it was discarded after the initial research phase. A paper outlining these issues was presented at the 1985 American Control Conference (see Appendix A). The minimum error excitation suboptimal design was successful and is advocated as the main design approach.

Exploratory research on nonlinear maneuver autopilot synthesis brought out the feasibility of generating the maneuver autopilots by employing singular perturbation theory in conjunction with prelinearizing transforms. The methodology is outlined, though the controllers have not been worked out. A simple example problem to serve as an illustrative example is given in Appendix B. This technique appears to hold considerable promise and will be further pursued in future research.

The designs obtained were scheduled as functions of Mach number and load factor and were tested on a linear simulation. The performance has been found satisfactory within the validity of the linear model assumptions. In the next phase these designs will be tested on a nonlinear simulation of the F-15 aircraft.

### 1.3 STUDY RESULTS

Results from this work fall into four categories, (1) control design technique evaluations, (2) specific control analysis and design, (3) software developments for control law mechanization, and (4) specific control law validations. The various deliverables in these areas are:

- 1) For control design technique evaluations
  - a) A paper evaluating the eigenstructure assignment technique [6]
  - b) An assessment and extension of the nonlinear prelinearizing control technique [7], and demonstration on a simple example. (See Appendix B)

- 2) For control analysis and design
  - a) Development and clarification of maneuver modeling equations beyond the analysis in references [4, 10].
  - b) Condensation of the F-15 aircraft nonlinear characteristics to a table of reference states and controls via nonlinear simulation trim values at approximately 145 conditions on the flight envelope.
  - c) Decomposition of the 8 maneuvers over the flight envelope into 30 linear perturbation models (15 with fixed throttle and 15 with variable thrust).
  - d) Solution of 30 output feedback gains which can be used with the 30 linear perturbation models to simulate maneuvers throughout the envelope.
- 3) For software developments
  - a) Extensions to Integrated Systems, Inc. (ISI's) MATRIX<sup>TM</sup> SYSTEM BUILD<sup>TM</sup> package including a linear time varying FORTRAN block to give a generic linear time varying simulation capability (such a capability can be easily mechanized to model linear time-varying simulations of engine and aircraft dynamics, for example).
  - b) Development of a three-dimensional interpolation program which converts table look-ups in altitude, Mach, and load factor to a one-dimensional table look-up with respect to time for a specific maneuver,
  - c) Documented command files Aircraft-CAS model building and Maneuver Auto Pilot (MAP) design in the MATRIX<sup>x</sup> language for the model generation, control law design, and simulation validation process, and
- 4) Demonstration of the maneuver autopilot validations in a linear simulation.

The linear control laws developed in this work are now ready for validation on a nonlinear batch simulation. With suitable algebraic equation manipulation, the nonlinear control laws can also be mechanized on a nonlinear simulation.

## 1.4 REPORT ORGANIZATION

Section 2 describes the aircraft and command augmentation system (CAS) models, the choice of outputs useful for feedback and the procedure for obtaining linear models. Section 3 specifies the flight test maneuvers analyzed in terms of the maneuver objective and how that objective can be reduced to a set of equations which constrain a required set of outputs. Section 4 outlines the linear techniques evaluated and used for the design of perturbation feedback controllers as well as the nonlinear tracking feedback controller. Section 5 reviews the linear simulation mechanization and discusses the demonstration results for the eight required maneuvers. Conclusions are given in Section 6. The appendices describe in detail the issues in linear time-varying simulation, the evaluation of constrained eigenstructure assignment and the output error feedback controller designs, and nonlinear tracking control via prelinearizing transformations.



SECTION 2  
AIRCRAFT AND COMMAND AUGMENTATION SYSTEM (CAS) MODELS

This section reviews the overall linear system model used to both develop and validate the linear perturbation control laws. Figure 2-1 below shows the design process used.

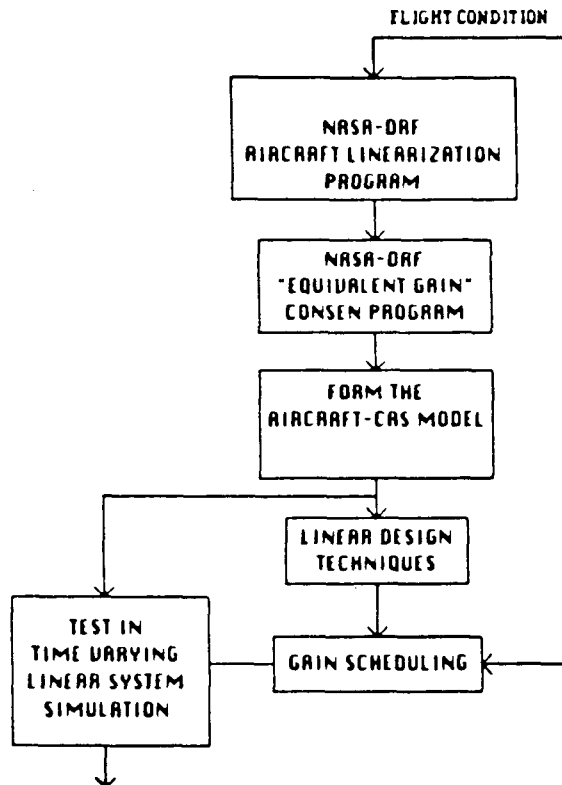


Figure 2-1. Use of Linear System Flight Test Trajectory Model

The rest of the section describes the airframe, engine, and command augmentation (CAS) models and how they are integrated into the overall linear design and evaluation model.

## 2.1 AIRCRAFT MODEL

As shown in Figure 2-1, the airframe model was obtained from the NASA Ames Dryden Flight Research Facility (ADFRF) LINEAR Program [11], yielding the aircraft model in the standard form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu},$$

$$\mathbf{y} = \mathbf{Hx} + \mathbf{Fu}$$

where

$$\mathbf{x}^T: [\delta V, \delta \alpha, \delta q, \delta \theta, \delta \beta, \delta p, \delta r, \delta \phi, \delta h],$$

$$\mathbf{y}^T: [\delta \dot{p}, \delta A_n, \delta q, \delta \dot{q}, \delta p, \delta A_{ny,i}, \delta \dot{r}, \delta r, \delta M, \delta \alpha, \delta \gamma, \delta \phi, \delta \beta].$$

The first nine outputs are required in the linearized CAS model.

## 2.2 ENGINE MODEL

An engine model of the form

$$\frac{0.2}{s + 0.2}$$

was assumed. However, since this lag can be compensated for with a simple lead-lag compensator, the problem of a slow engine time constant was not

included in the overall design, assuming exact cancellation. The actual residual due to mismodeling can best be addressed with the nonlinear simulation. Figure 2-2 below shows the overall mechanization including the CAS states described next.

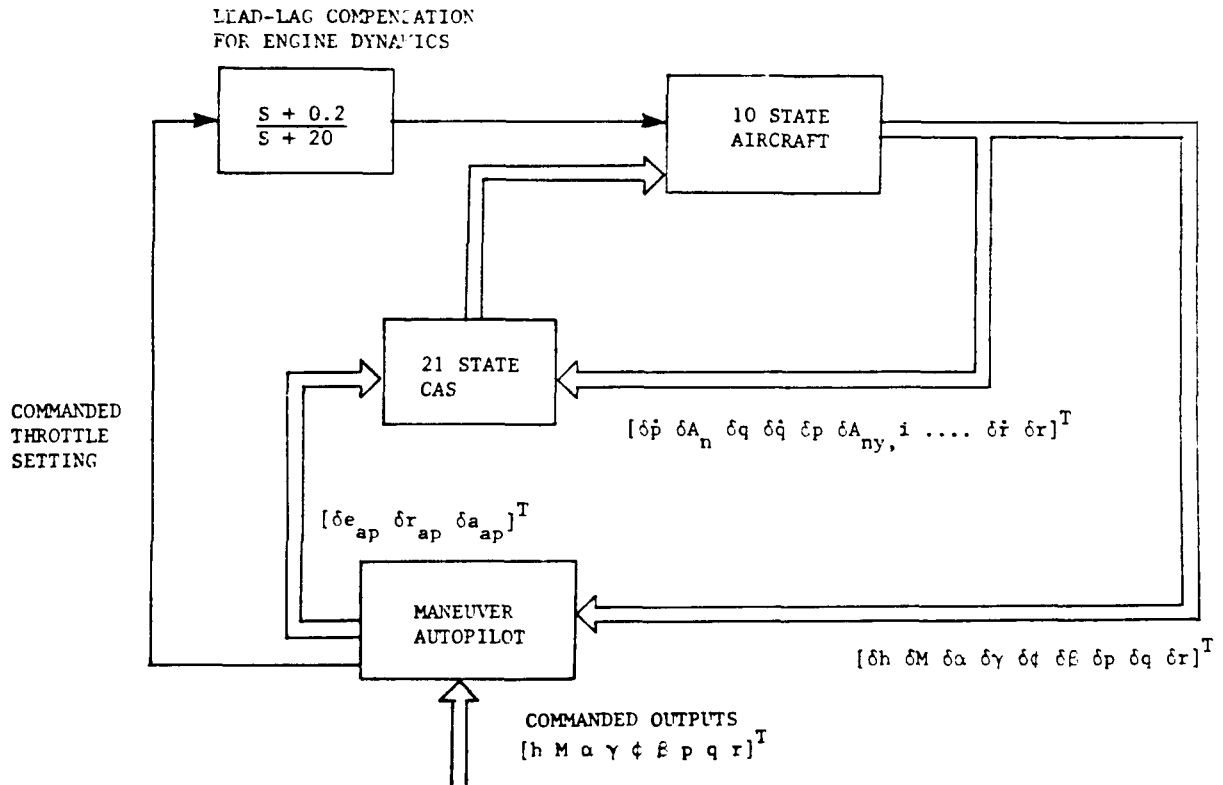


Figure 2-2. Linear Perturbation Flight Test Trajectory Control System

### 2.3 COMMAND AUGMENTATION SYSTEM (CAS) MODEL

The CAS model is highly nonlinear with gain schedules and multiplicative and saturation nonlinearities. This was linearized with a gross linearization and an "equivalent gain" for the multiplicative nonlinearities determined by the NASA ADFRF CONSEN program. The CONSEN program simulates the actual CAS for several time steps at a given flight condition. When all the transients have decayed, the ratio of inputs and outputs are then used to compute the equivalent gains.

The CAS system has no access to ailerons in the aircraft under consideration and hence, all the flight test maneuvers will be accomplished using throttle, elevator, rudder and differential tail. The maneuver autopilot outputs will be connected to the existing autopilot interface in the CAS, as shown in Figures 2-3 through 2-5.

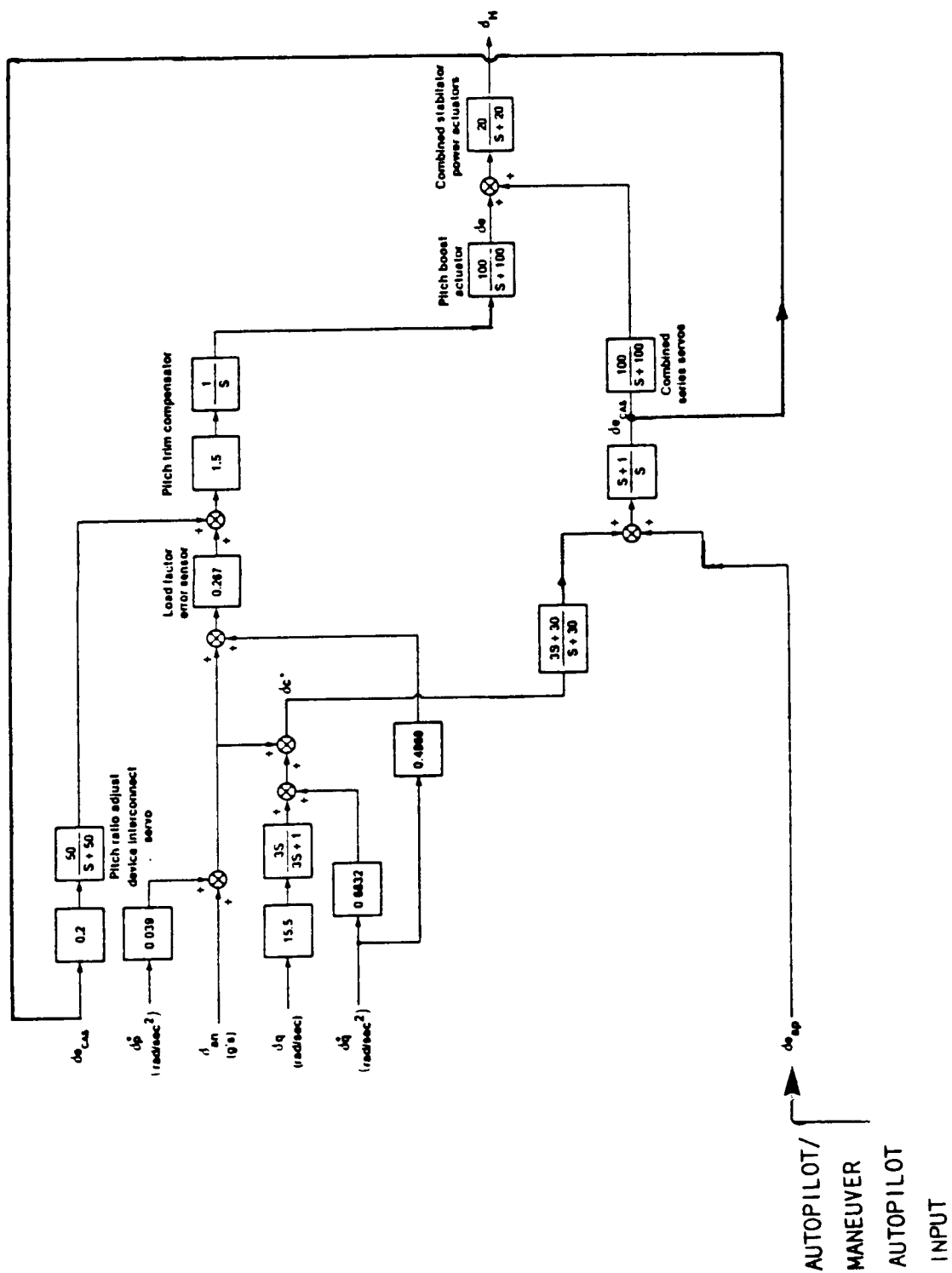


Figure 2-3. Pitch CAS - Gross Linearized System  
(Pilot Inputs and Trim Loops Eliminated)

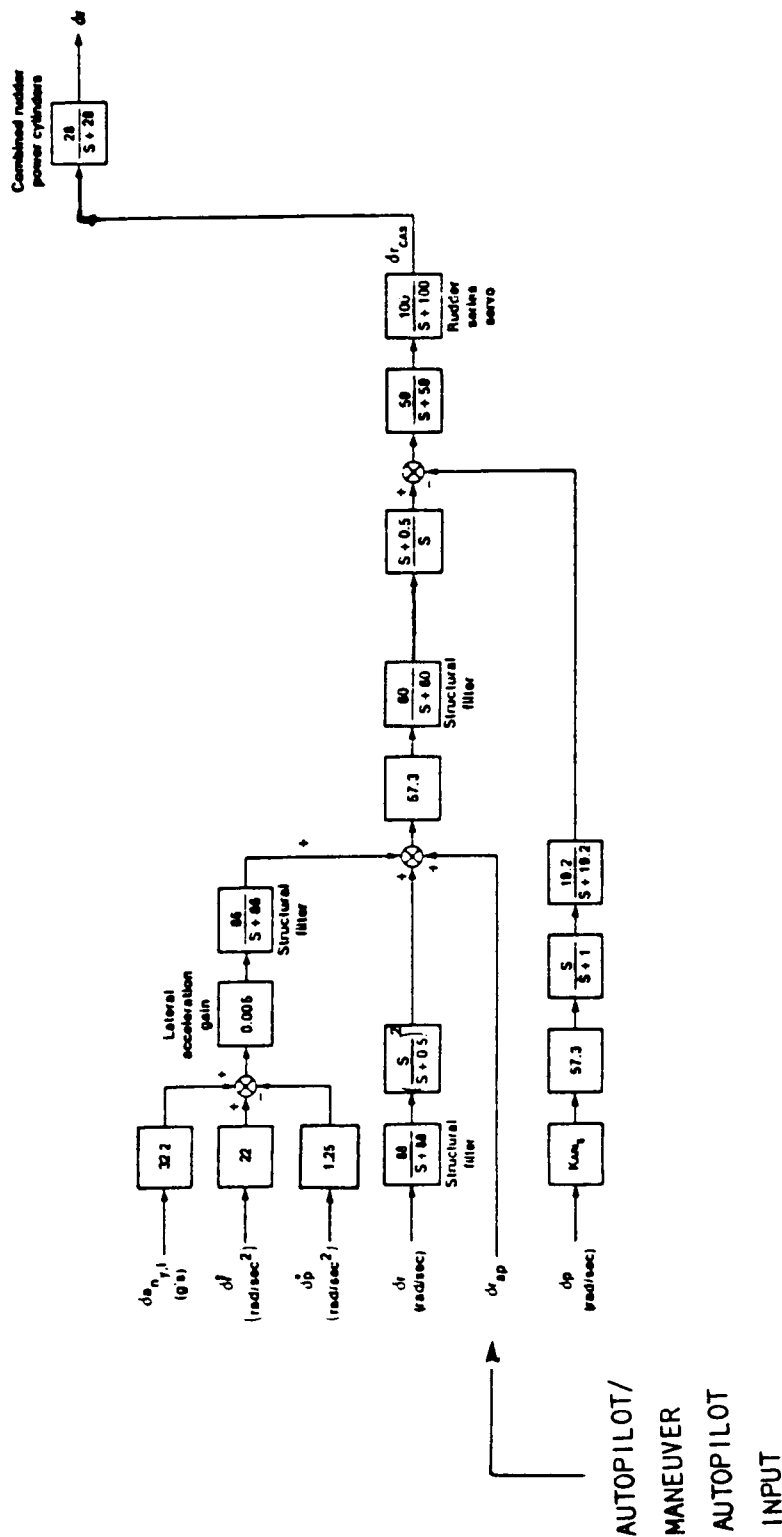


Figure 2-4. Yaw CAS Gross Linearized System  
 (Pilot Inputs and Trim Loops Eliminated Gain  
 Kari<sub>s</sub> determined from "CONSEN" Program)

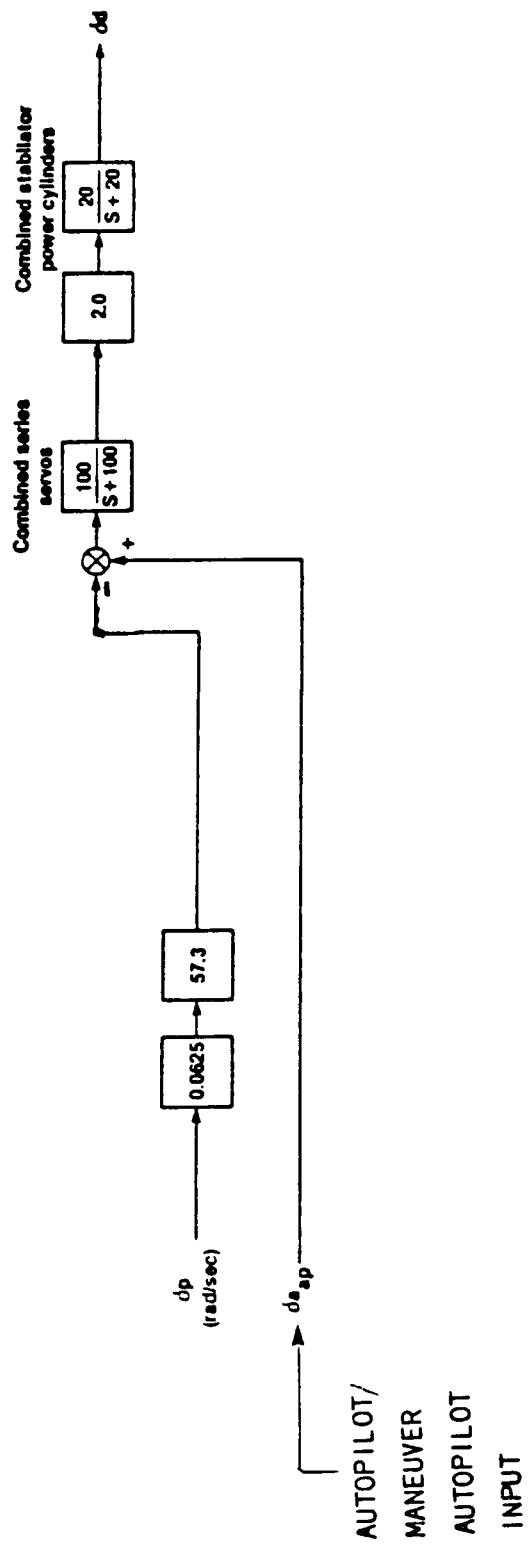


Figure 2-5. Roll CAS Gross Linearized System  
(Pilot Inputs and Trim Loops Eliminated)

## 2.4 LINEAR MODEL SUMMARY

The overall linear model has nine aircraft states, one engine state, and twenty-one CAS states. The outputs to be used in the maneuver autopilot, for all maneuvers, are

$$\mathbf{y}^T = [\delta h, \delta M, \delta \alpha, \delta \gamma, \delta \phi, \delta \beta, \delta p, \delta q, \delta r].$$

Selection of these quantities for feedback are primarily dictated by the various flight test maneuvers to be executed. Thus, the altitude, Mach number and the angle of attack are to be either regulated about reference values or should be made to follow desired command histories. The choice of roll attitude feedback arises from a requirement to maintain wings level along symmetric flight test trajectories. In nonsymmetric flight test maneuvers, desired load factors can be sustained by an appropriate roll attitude command. For all flight test maneuvers, the angle of sideslip needs to be regulated about zero; this prompts the use of  $\beta$  feedback. An output feedback controller is envisaged in the present development. Since most output feedback synthesis approaches do not guarantee stability, it is decided to incorporate derivative feedbacks for as many output variables as feasible. This led to the selection of the body rates  $p$ ,  $q$ ,  $r$  to serve as derivative feedbacks for angle of attack, angle of sideslip and the roll attitude. The Mach number and the flight path angle serve as derivative feedbacks for altitude. Since most flight test maneuvers require angle of attack tracking, an integral feedback is incorporated in the angle of attack channel. While executing nonsymmetric flight test maneuvers, the roll attitude can sometimes be very close to  $90^\circ$ . At these conditions, small perturbations in the roll attitude can induce major changes in altitude and other states. Hence in order to provide a tight roll attitude control, two integral feedbacks are introduced for the roll attitude  $\phi$ .



## SECTION 3

### MANEUVER MODELING

The desired trajectory or maneuver to be modeled governs the subsequent control law development in two ways. First, the maneuver prescribes the equations of motion of the vehicle on the reference trajectory and secondly the maneuver regimes determine the linear perturbation equations about the commanded trajectory from which the linear control law is developed.

Aircraft flight test trajectories could be based on inertial reference (e.g., level-turn or 3-D guidance) or reference with respect to another vehicle or vehicles (e.g., in air-to-air combat). To place the flight test trajectory control design problem in a proper framework, the constraints which determine the equations of motion for various large classes of trajectories are described below.

The subsections which follow give a general overview of maneuver modeling and the descriptions of the specified maneuvers analyzed in this work.

#### 3.1 MANEUVER MODELING OVERVIEW

We divide single vehicle flight paths into those which require continuous control along the trajectory and those that specify a final flight condition. In either case the flight test trajectory could be specified in terms of one of the following:

1. Constraints on position components,
2. Constraints on velocity components and altitude,
3. Constraints on combinations of load, speed and altitude.

Combinations of these constraints could also be considered. The flight test maneuvers discussed in this report belong to the second and third categories. Reference [4] gives some early results in flight test trajectory modeling.

### 3.1.1 Constraints on Position Components

Examples of trajectories which involve position constraints along the flight path are

1. 4-D guidance ( $x(t)$ ,  $y(t)$ ,  $h(t)$  are given functions of time).
2. 3-D guidance ( $x$ ,  $y$ ,  $h$  are related to each other, e.g., fly along a hypothetical wire in space). Examples of 3-D guidance are approach to landing, terminal area flight paths and threat evasion for reconnaissance aircraft and bombers. This also includes straight and level flight and flights along predetermined paths.

Examples of trajectories which specify position constraints at the final trajectory point are:

1. 4-D specification (arrive at a certain point, at a certain time, e.g., touchdown on the runway at a specified point).
2. 3-D specification (fly-to-VOR, terrain following).

Note that each of these trajectories requires position measurement. The 4-D guidance trajectory indirectly specifies velocity and acceleration components. Thus, specification of position components is the most comprehensive constraint on the trajectory. Such a rigid constraint may be unnecessary for most test maneuvers.

### 3.1.2 Constraints on Velocity Components and Altitude

While the horizontal position components do not, in general, affect aerodynamic variables, the altitude determines density and by itself affects dynamic pressure. Thus, it must always be considered as a possible variable

to be constrained. In fact, the altitude and dynamic pressure are so important that a majority of flight test trajectories will define the altitude profile (this includes maintaining constant altitude).

Examples of this class of trajectories are:

1.  $u(t)$ ,  $v(t)$ ,  $w(t)$  and  $h(t)$  [in other words, Mach number, dynamic pressure,  $\beta(t)$  and  $\alpha(t)$ .]  $\beta(t)$  may be zero.
2. Mach number, angle-of-attack and dynamic pressure (as in shuttle tile tests).

Various other combinations of velocity components and altitudes could also be specified.

Mach number, angle-of-attack and altitude constraints could also be desired at one point on the trajectory. For example, the zoom-and-pushover is a trajectory where angle-of-attack, Mach number and altitude are specified at one point on the trajectory.

### 3.1.3 Constraints on Combinations of Load, Speed and Altitude

The trajectory specifications could involve components of loads along the three axes, velocity components and altitude. The typical load specification will consist of desired vertical acceleration. The desired value of the lateral acceleration is usually zero. The total speed is often specified in lieu of the fore-and-aft acceleration.

Many combinations of load, speed and altitude specifications are possible. Some examples are as follows:

1. A constant load, constant Mach number level turn,
2. A constant Mach number, constant altitude windup turn.

Often, the desired flight trajectory for an aircraft depends upon the position and flight test trajectory of other vehicles. Typical examples are collision avoidance, air combat, or avoidance of air-to-air missiles. The specification is typically based on the position of a target aircraft with respect to the aircraft whose trajectory is being controlled.

The next section gives a specific objectives and their analytical development for each of the maneuvers for which autopilots were designed.

### 3.2 MANEUVER MODELING

To summarize, the objective of maneuver modeling is to generate a consistent set-of state and control histories to serve as commands and open loop controls for the maneuver autopilot using a data base consisting of trim conditions. Two sets of trims have been found adequate for all the maneuvers discussed here, viz, straight and level trims and level turn trims. To the extent feasible, kinematic relationships are employed to generate the state-control histories. Whenever this is not possible, linearized aerodynamics and engine models are used. Note that the following development is not restricted a particular aircraft.

The commands and the open loop controls consist of:

Commands: Altitude, Mach number, angle-of-attack, flight path angle, roll attitude, angle of side slip, roll-pitch-yaw body rates.

Openloop Controls: Throttle, elevator, rudder and differential tail.

In the following, the maneuver modeling for individual flight test trajectories will be discussed in detail. It is important to note that all these maneuvers assume zero sideslip.

### 3.2.1 Transient Trajectory

The objective of this maneuver is to transfer an aircraft flying straight and level at a Mach-altitude pair to another Mach-altitude pair at a desired flight path angle. This maneuver is normally employed as the initial-terminal transient to other flight test maneuvers and hence the name.

The simplest way to mechanize this maneuver to assume a cubic polynomial in time for the altitude. Thus,

$$h = h_o + a_1 t + a_2 t^2 + a_3 t^3; t_o \leq t \leq t_f \quad (3.1)$$

from which

$$\dot{h} = a_1 + 2a_2 t + 3a_3 t^2 \quad (3.2)$$

The requirement for a cubic polynomial arises from the constraints that one wishes to place at the two ends, i.e., initial and final altitudes are specified along with altitude rates at the two boundaries. To simplify the development, constant acceleration along the flight path is assumed next, viz,

$$V = V_o + \dot{V}t, \quad (3.3)$$

where

$$\dot{V} = \frac{V_f - V_o}{t_f}, \quad V_f \text{ is the specified final speed and } t_f, \text{ the final time.}$$

The flight path angle is readily computed from

$$\gamma = \sin^{-1} \left( \frac{\dot{h}}{V} \right)$$

As noted elsewhere, a data base consisting of straight and level trims at several Mach-altitude pairs are available.

Assuming that the path angle is small, such that

$$\text{Lift} \approx \text{weight}$$

along this maneuver, the angle of attack history can be generated by linearly interpolating between stored straight and level trim data at the Mach number - altitude given by the equations (3.1) and (3.3). Since the angle of attack will be close to the straight and level trim values, the aerodynamic surface setting at these trims can be used to generate approximate open loop control settings. Assuming next that under these conditions, the actual drag is close to the trim values, the required thrust may be computed as follows.

Assuming that the aircraft thrust is aligned with the vehicle longitudinal axis, the acceleration along the flight path for symmetric flight is given by

$$\dot{V} = \frac{T \cos \alpha_{\text{trim}} - D}{m} - g \sin \gamma \quad (3.4)$$

In the expression (3.4), T is the thrust,  $\gamma$  flight path angle,  $\alpha$  the angle of attack, D the aerodynamic drag, m the aircraft mass and V the velocity along the flight path. In order to compute the required thrust, the equation (3.4) can be manipulated to the form given below

$$T = \frac{(\dot{V} + g \sin \gamma)m + D}{\cos \alpha_{\text{trim}}} \quad (3.5)$$

To compute the throttle setting, a linear thrust-throttle characteristic will be assumed. Thus,

$$T_{\max} = \frac{T_{\text{trim}}}{\eta_{\text{trim}}}, \quad (3.6)$$

$\eta_{\text{trim}}$  is the straight and level throttle setting and  $T_{\max}$  is the thrust corresponding to maximum throttle setting.

$$\therefore \eta_{\text{actual}} = T/T_{\max} \quad (3.7)$$

During this maneuver, one expects the body rates to be small. Consequently, the commanded values of these quantities are zero.

If the throttle setting emerging from this analysis is greater than the maximum or less than the minimum, it indicates that the assumed time of flight for the maneuver is unrealistic or that the model is inadequate or both. This quantity has then to be changed appropriately to make the maneuver feasible.

### 3.2.2 Level Acceleration/Deceleration

This is a wings level, constant altitude maneuver with Mach number constant or changing at a specified rate. The maneuver modeling for this trajectory is essentially a subset of the trajectory 3.2.1. Putting  $a_1=a_2=a_3=0$  in (3.1) results in

$$\begin{aligned} h &= h_0, \text{ constant} \\ \gamma &= 0, \text{ constant} \end{aligned} \quad (3.8)$$

and  $V = V_0 + \dot{V}t$ , with  $\dot{V}$  specified.

The required thrust and the corresponding throttle can be computed as in 3.2.1. The open loop control surface settings and the other commands are linearly interpolated from the straight and level trim data base.

### 3.2.3 Pushover, Pullup

This is a wings level, constant Mach number maneuver in which the angle of attack is varied a specified increment about the trim value at some specified rate. The maneuver is shown in Fig. 3.1. The corresponding angle of attack history is given in Fig. 3.2.

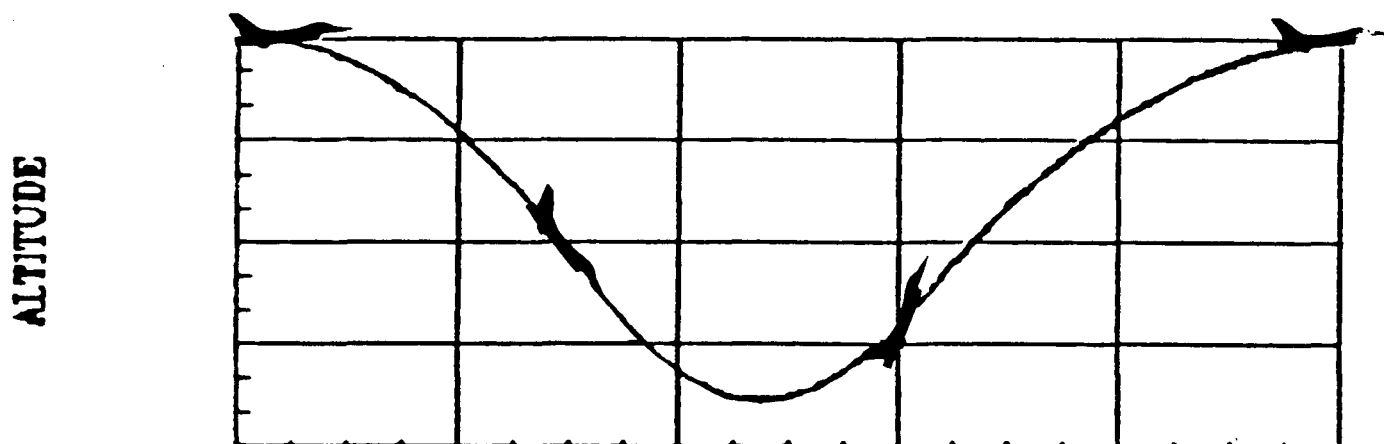


Figure 3.1. Pushover - Pullup Flight Test Trajectory.



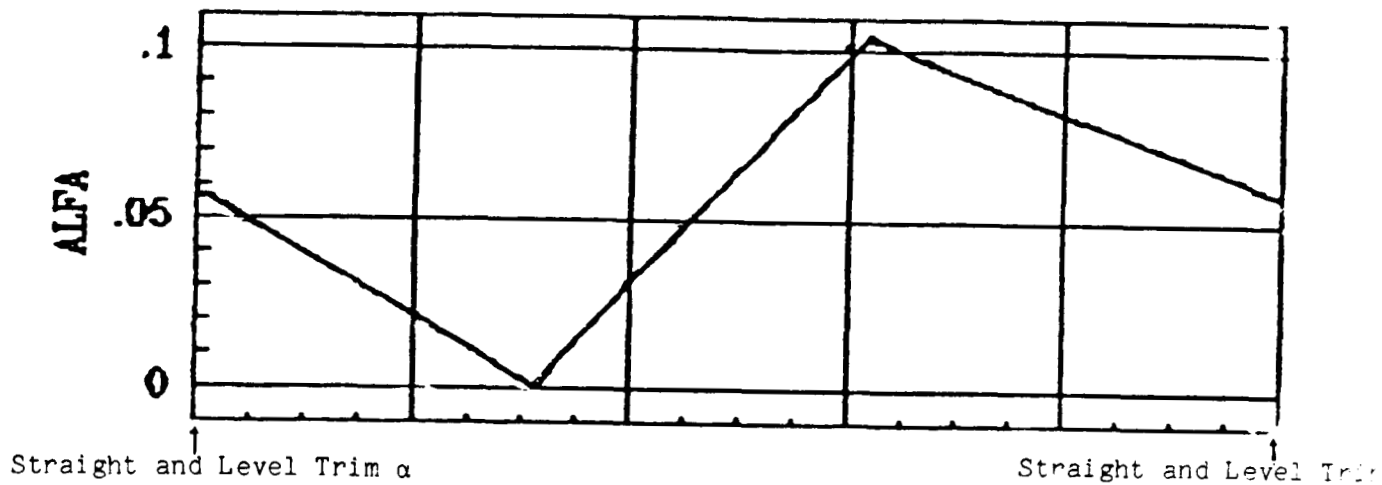


Figure 3.2. Angle of Attack History Along the Pushover - Pullup Flight Test Trajectory.

The maneuver modeling for this flight test trajectory uses the assumption that pitch rate is close to zero. Since the aircraft is in symmetric flight, the flight path angle  $\gamma$  can be calculated as the difference between the pitch attitude and  $\theta$  and the angle of attack  $\alpha$ . Thus

$$\gamma = \theta - \alpha, \quad (3.9)$$

and  $\alpha$  is specified as a function of time, the flight path angle can be computed. Now,

$$\dot{h} = V \sin \gamma = MC \sin \gamma \quad (3.10)$$

where  $C$  is the speed of sound specified as a function of altitude  $h$  and  $M$  is the desired Mach number. Equation (3.10) can be analytically or numerically integrated to yield the altitude history. The throttle setting may be computed from the following equations.

Since Mach number is to remain constant throughout the maneuver, one can differentiate the expression for Mach number ( $M = \frac{V}{C}$ ) with respect to time and equate to zero to obtain

$$\frac{\dot{V}}{C} - \frac{V}{C^2} \frac{\partial C}{\partial h} \dot{h} = 0 \quad (3.11)$$

substituting for  $\dot{h}$  from (3.10), one has

$$\dot{V} = \frac{V^2}{C} \frac{\partial C}{\partial h} \sin \gamma \quad (3.12)$$

Equating expressions (3.12) and (3.4), one has

$$\frac{V^2}{C} \frac{\partial C}{\partial h} \sin \gamma = \frac{T \cos \alpha - D}{m} - g \sin \gamma \quad (3.13)$$

From which

$$T = \left[ \frac{V^2}{C} \frac{\partial C}{\partial h} + g \right] \frac{m \sin \gamma}{\cos \alpha} + \frac{D}{\cos \alpha} \quad (3.14)$$

The drag at the commanded angle of attack can be computed from linearized drag coefficient specified as a function of  $\alpha$ , the dynamic pressure and the reference area. As before, the linear throttle assumption is invoked to compute the throttle setting.

$$\eta_{\text{actual}} = T \cdot \frac{\eta_{\text{trim}}}{T_{\text{trim}}} \quad (3.15)$$

#### 3.2.4 Zoom and Pushover

The zoom and pushover is a wings-level, thrust stabilized less than 1g maneuver. The flight trajectory is a parabolic path with the target Mach/altitude/angle of attack point at the apex. An illustration of the zoom and pushover flight test trajectory is given in Fig. 3.3. The maneuver begins at O with straight and level flight conditions. A transient-maneuver is performed to transfer the aircraft to the point A with all controls active. At the point A, the throttle is fixed at a predetermined value and the aircraft executes the zoom and pushover trajectory. At the point B, the thrust control is reinstated and a transient trajectory transfers the aircraft back to straight and level flight conditions at point C.

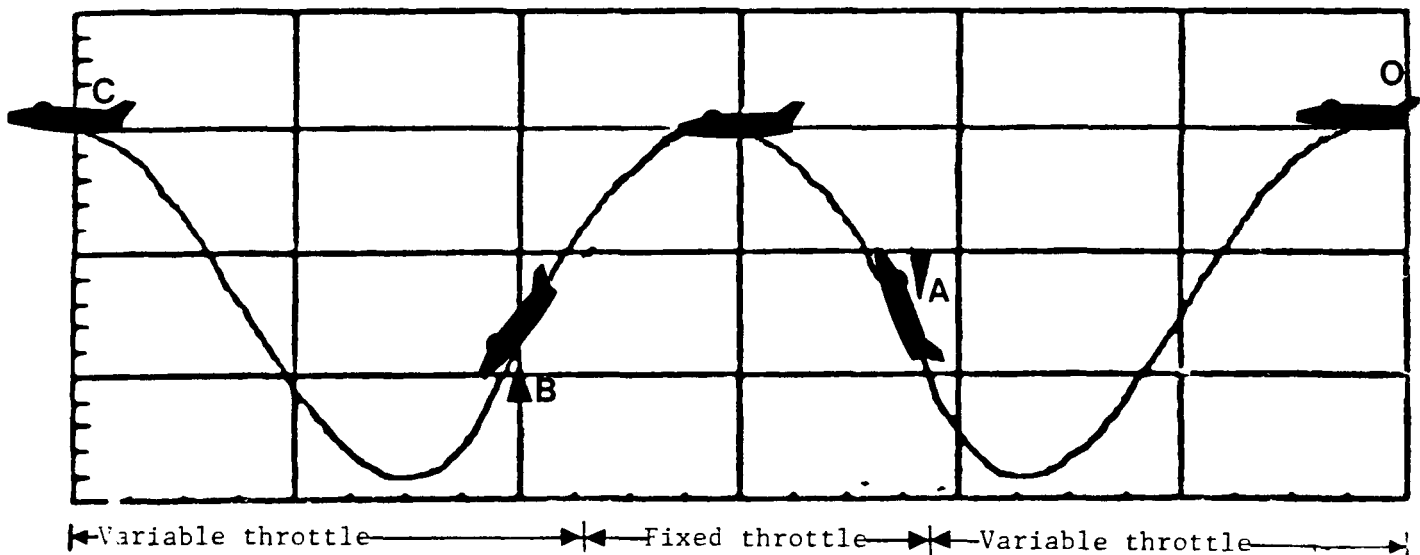


Figure 3.3. Zoom and Pushover Flight Test Trajectory.

This is perhaps the most complicated of all the symmetric flight test maneuvers. This flight test trajectory consists of three phases. In the first phase, the aircraft is transferred from its straight and level flight condition to the beginning of the parabolic flight path. The second phase is the required zoom and pushover maneuver followed by the third phase which brings the aircraft back to its original straight and level flight conditions. The first and third phase maneuvers are essentially transients and the maneuver modeling discussed in 3.2.1 is directly applicable. The second phase will be analyzed in this section.

A parabolic flight path has the following properties

1. Horizontal acceleration is zero
2. Vertical acceleration is constant
3. Total energy is constant.

Since the apex speed is specified, say  $V_T$ , one has

$$\dot{x} = V \cos \gamma = \text{constant} = V_T$$

Here,  $x$  is the down range. Note that since the aircraft is in symmetric flight, the cross range is zero. Thus,

$$\gamma = \pm \cos^{-1} \left[ \frac{V_T}{V} \right] \quad (3.16)$$

a positive or negative sign has to be chosen based on whether the aircraft is flying towards the apex or away from it.

From the given apex speed, angle of attack and altitude, the lift and drag can be computed using the straight and level trims data base using

linearized aerodynamic coefficients. From constant energy property, one has, at the apex of the parabola,

$$T \cos \alpha_T = D \quad (3.17)$$

From which, the throttle setting at the apex can be computed as follows

$$\eta_{\text{actual}} = T \cdot \frac{\eta_{\text{trim}}}{T_{\text{trim}}} \quad (3.18)$$

with lift and thrust, the vertical acceleration at the apex can be computed. Thus

$$\frac{T \sin \alpha_T + L}{m} - g = g_a \quad (3.19)$$

Note that  $g_a$  should be a negative quantity, numerically less than the acceleration due to gravity  $g$ . The acceleration  $g_a$  has to remain constant through the parabolic path. To summarize, the aircraft trajectory approximates the path of a projectile in a uniform conservative force field. The total energy of the aircraft in this field is given by

$$E = h + \frac{v^2}{-2g_a} = \text{constant} \quad (3.20)$$

Expression (3.20) can be used to compute the speed along the parabolic path, given the altitude.

Next, given the altitude at which the parabola is to begin, and perhaps end, one can write

$$h = h_o + \dot{h}_o t + \frac{g_a}{2} t^2 \quad (3.21)$$

with  $\dot{h}_o$  computed using the following relations.

$$V_o = \sqrt{-2g_a(E-h_o)}$$

$$\gamma_o = \cos^{-1} \left[ \frac{V_T}{V_o} \right]$$

and

$$\dot{h}_o = V_o \sin \gamma_o \quad (3.22)$$

The time of flight on this parabola is easily computed with equation (3.21) in the general case or with the following equation if the zoom and pushover parabola is symmetric about its axis.

$$t_f = \frac{2}{-g_a} \sqrt{(V_o^2 - V_T^2)} \quad (3.23)$$

Note that the throttle is to remain fixed at the value given by the equation (3.18). The angle of attack throughout the parabolic path is computed from

$$\frac{T \cdot \alpha + L_o + L_\alpha \alpha}{m} - g \cos \gamma = g_a \quad (3.24)$$

with  $T = T_{\max} \cdot \eta_{\text{actual}}$  and small angle approximation for  $\alpha$  has been used.

The open loop control surface settings are assumed to be the interpolated values from the straight and level trim data base.

### 3.2.5 Excess Thrust Windup Turn

This is a maneuver with angle of attack linearly increasing from the wings-level trim condition to some specified final value at a specified rate. The maneuver is performed at constant altitude and constant Mach number. A schematic figure of this maneuver is shown in Fig. 3.4.

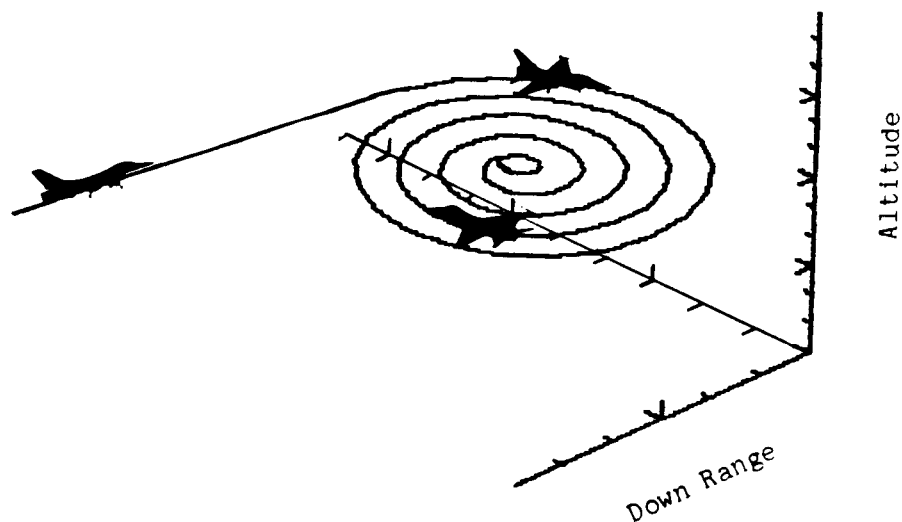


Figure 3.4. Excess Thrust Windup Turn Flight Test Trajectory.

As mentioned elsewhere in this report, the trim data base consists of straight and level trims at several Mach-altitude pairs along with level

turns at several Mach-altitude-load factor conditions. Since excess thrust windup turn can be considered to consist of several level turn trim conditions appropriately pieced together, this maneuver model is merely a 3-Dimensional interpolation using the trim data base.

### 3.2.6 Constant Throttle Windup Turn

This is a maneuver with angle of attack increasing linearly at a specified rate from trim to some specified final value. The maneuver is performed at a predetermined, constant thrust level. Mach number is maintained by trading potential for kinetic energy via an appropriate altitude rate.

Since Mach number is constant, one can write as in equation (3.12)

$$\dot{V} = \frac{V^2}{C} \frac{\partial C}{\partial h} \sin \gamma \quad (3.25)$$

Further, since the altitude, Mach number and angle of attack at the initial point are known, one can compute

$$T_{\max} = T_{\text{trim}} / \eta_{\text{trim}} \quad (3.26)$$

from the trim data base. Let the throttle be fixed at a value  $\eta_R$ . Thus, the actual thrust

$$T_R = T_{\max} \cdot \eta_R \quad (3.27)$$



If the actual thrust  $T_R$  is greater than that required for level turn at the given Mach-altitude-angle of attack condition, the Mach number can be maintained constant only by a positive altitude rate. The reverse applies whenever  $T_R$  is less than  $T_{trim}$ . Let the excess thrust over the level turn trim be

$$\Delta T = T_R - T_{trim} \quad (3.28)$$

Now, one has

$$\frac{V^2}{C} \frac{\partial C}{\partial h} \sin \gamma = \frac{\Delta T \cdot \cos \alpha}{m} - g \sin \gamma \quad (3.29)$$

from which,

$$\gamma = \sin^{-1} \left[ \frac{\Delta T \cos \alpha}{m \left( \frac{V^2}{C} \frac{\partial C}{\partial h} + g \right)} \right] \quad (3.30)$$

Also,

$$\dot{h} = V \sin \gamma \quad (3.31)$$

The expression (3.31) can be numerically integrated over one step to obtain the new altitude. The calculations may be repeated as many times as one wishes to obtain the trajectory.

It is important to begin this trajectory at a high-g turn, since fixing the throttle at a straight and level condition can result in an initial acceleration or a high path angle climb/descent. Hence, in order to avoid confusion, this maneuver requires an initial and terminal maneuver. Thus three phases are required to execute this maneuver.

1. A trajectory beginning at straight and level flight at the desired Mach-altitude pair and ending at a high-g level turn with all control surfaces and throttle active.
2. Constant thrust windup trajectory.
3. Terminal maneuver at constant altitude transferring the vehicle from the level turn at constant thrust conditions to straight and level flight.

A typical constant thrust windup turn trajectory is given in Fig. 3.5.

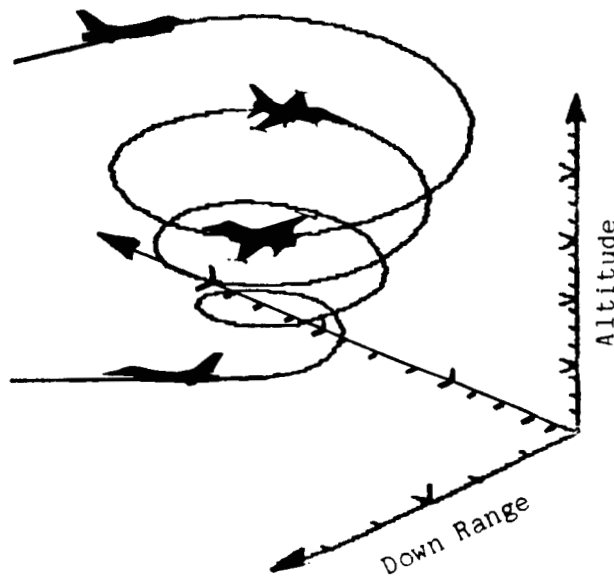


Figure 3.5. Constant Thrust Windup Turn Trajectory Descending Flight.

### 3.2.7 Constant Dynamic Pressure and Constant Load Factor Trajectory

This maneuver is initiated at a predetermined load factor, Mach number, and dynamic pressure. Thus, the initiation of this maneuver is not necessarily the wings-level condition. This maneuver can be either an ascending or descending at a specified Mach number rate. Dynamic pressure and load factor are held constant throughout the maneuver. Altitude is gained or lost to maintain dynamic pressure with changing Mach number.

As in the maneuver 3.2.6, this trajectory also requires three phases. The first is a level turn to achieve the required load factor from straight and level flight at the desired Mach-altitude pair. The third phase restores the aircraft to straight and level flight at the final Mach-altitude condition. Both these maneuvers can be constructed by interpolating between the stored trim data points at the current Mach-altitude-load factor conditions. In the following we shall discuss the constant dynamic pressure-constant load factor trajectory modeling. Note that this development does not depend explicitly on the load factor. Thus, this model is valid for all load factors including a constant unity load factor, wings level, constant dynamic pressure trajectory.

The dynamic pressure,  $Q$  is given by

$$Q = \frac{1}{2} \rho(h) \cdot V^2 \quad (3.32)$$

Differentiating the expression (3.32) with respect to time and using the altitude rate equation, with  $\dot{Q} = 0$ , one has

$$\frac{1}{2} \frac{\partial \rho}{\partial h} V^3 \sin \gamma + \rho(h) \dot{V} = 0$$

or

$$\dot{V} = -\frac{1}{2\rho} \frac{\partial \rho}{\partial h} V^2 \sin \gamma \quad (3.33)$$

Since the Mach number is given by

$$M = V/C(h),$$

in order to maintain a desired Mach rate, one must have

$$\dot{M} = \frac{\dot{V}}{C} - \frac{V^2}{C^2} \frac{\partial C}{\partial h} \sin \gamma$$

from which

$$\dot{V} = \dot{M} C + \frac{V^2}{C} \frac{\partial C}{\partial h} \sin \gamma \quad (3.34)$$

Equating the expressions (3.33) and (3.34), one has

$$\gamma = \sin^{-1} \left[ \frac{-\dot{M} C}{\left( \frac{1}{2\rho} \frac{\partial \rho}{\partial h} + \frac{1}{C} \frac{\partial C}{\partial h} \right) V^2} \right] \quad (3.35)$$

There are two important conclusions that can be drawn from the expression (3.35)

1. Since  $\frac{\partial \rho}{\partial h}$  and  $\frac{\partial c}{\partial h}$  are negative (density and speed of sound decrease with increasing altitude), a positive  $\dot{M}$  will result in a climbing trajectory while a negative  $\dot{M}$  will yield a descending path.
2. This maneuver cannot be flown at altitudes where  $\frac{\partial \rho}{\partial h}$  and  $\frac{\partial c}{\partial h}$  are nearly zero unless the desired Mach rate is also zero.

As before, the altitude rate equation

$$\dot{h} = V \sin \gamma \quad (3.36)$$

may be numerically integrated over a small time step to obtain the new altitude. The actual throttle setting is again computed from the level turn trim thrust and throttle setting at the current Mach-altitude-load factor as follows.

$$T_{\max} = T_{\text{trim}} / \eta_{\text{trim}} \quad (3.37)$$

$$T = \left[ g - \frac{1}{2\rho} \frac{\partial \rho}{\partial h} V^2 \right] \frac{m \sin \gamma}{\cos \alpha} + \frac{D}{\cos \alpha} \quad (3.38)$$

$$\eta_{\text{actual}} = T / T_{\max} \quad (3.39)$$

The angle of attack and drag in the expression (3.38) are the interpolated values from the trim data base at the current Mach-altitude-load factor. The Mach number as a function of time is obtained from

$$M = M_o + \dot{M}t \quad (3.40)$$

The command body rates and open loop control surface deflections are again the interpolated values from the trim data base.

### 3.2.8 Constant Reynolds Number and Constant Load Factor Trajectory

This maneuver is initiated at a predetermined load factor, Mach number and Reynolds number. Thus, the initiation of this maneuver is not necessarily the wings-level condition. This maneuver can be either ascending or descending at a specified Mach number rate. Reynolds number and load factor are held constant throughout the maneuver. Altitude is gained or lost to maintain Reynolds number with changing Mach number.

This maneuver model is different from 3.2.7 only in the way that one computes the flight path angle and the throttle setting. Hence only these two aspects will be discussed in the following. As in maneuver 3.2.7, the modeling does not depend explicitly on the load factor. Consequently, the following development is valid for all load factors including a wings level, unity load factor - Constant Reynold's number trajectory.

The Reynold's number,  $Re$  is given by

$$Re = \frac{VD\rho(h)}{\mu(h)}, \text{ where } \mu \text{ is the Viscosity of atmosphere} \quad (3.41)$$

Differentiating the expression (3.41) with respect to time and using the altitude rate equation, with  $\dot{R}_e = 0$ ; one has

$$\dot{V} = \left( \frac{1}{\mu} \frac{\partial \mu}{\partial h} - \frac{1}{\rho} \frac{\partial \rho}{\partial h} \right) V^2 \sin \gamma \quad (3.42)$$

Equating the expression (3.42) to equation (3.34),

$$\gamma = \sin^{-1} \left[ \frac{\dot{M}C}{\left( \frac{1}{\mu} \frac{\partial \mu}{\partial h} - \frac{1}{\rho} \frac{\partial \rho}{\partial h} - \frac{1}{C} \frac{\partial C}{\partial h} \right) V^2} \right] \quad (3.43)$$

As in the constant dynamic pressure, constant load factor flight test trajectory, we note that this maneuver cannot be flown at altitudes where  $\frac{\partial \mu}{\partial h}$ ,  $\frac{\partial \rho}{\partial h}$  and  $\frac{\partial C}{\partial h}$  are nearly zero unless the desired Mach rate is also zero. The actual throttle setting can be computed from the level turn trim thrust and throttle setting at the current Mach-altitude-load factor as follows.

$$T = \left[ \left( \frac{1}{\mu} \frac{\partial \mu}{\partial h} - \frac{1}{\rho} \frac{\partial \rho}{\partial h} \right) V^2 + g \right] \frac{m \sin \gamma}{\cos \alpha} + \frac{D}{\cos \alpha} \quad (3.44)$$

$$\eta_{\text{actual}} = T \frac{\eta_{\text{trim}}}{\eta_{\text{trim}}} \quad (3.45)$$

To facilitate easier computations, a FORTRAN program has been written to generate the required commands and open loop control settings given the appropriate data. A listing of this code is given in Appendix C.

## SECTION 4

### MANEUVER AUTOPILOT DESIGN

The previous section described the required maneuver modeling, whereby for eight chosen maneuvers, a subset of the outputs are constrained to prespecified time histories. For the control analysis and design done in this study to have any value when applied to the F-15 nonlinear simulation or the actual aircraft, consistent values for all of the dynamic states and corresponding control values along the trajectory must be found. There are at least two straightforward ways to generate the required reference states and controls. First, one could iteratively simulate with the nonlinear model until an open loop law approximates the desired output time histories. This could be done systematically with numerical nonlinear optimization, using a parameterization of the control surface and thrust time histories. Or secondly, one can trim the nonlinear simulation at a number of conditions close to the desired trajectory and approximate the dynamic reference trajectory from these trim values. The latter approach was chosen both because of the completeness and flexibility of this tabular representation of the nonlinear aircraft characteristics, and because of the connection with the perturbation trim controllers designed to work along with the reference trajectory commands. It should also be pointed out that this "static" approach eliminated the need for any nonlinear simulation during the control design stage, apart from execution of a linearizing program [11] which contains the nonlinear F-15 aircraft equations.

The next subsection shows how a table of trim values can be used to develop a "linear model" of the entire F-15 flight test system for design and evaluation of the aircraft dynamic response in specified nonlinear maneuvers. A second subsection outlines two different linear control design techniques, evaluating their strengths and weaknesses, giving perturbation controller designs using these two techniques. A final subsection presents an assessment and extension of the nonlinear prelinearizing control technique [12], the application of which will be demonstrated in the next study phase, along with linear gain scheduled controllers, on the full nonlinear dynamic simulation.



#### 4.1 THE F-15 FLIGHT TEST SYSTEM "LINEAR" MODEL

It must be emphasized that while the control design approaches demonstrated in this work are based on linear models, the maneuvers desired are highly nonlinear. As discussed above, the nonlinear characteristics of the F-15 are condensed into a table of trim values, which when properly used with linear perturbation controllers gives a linear time varying simulation which accurately represents the nonlinear aircraft response through the nonlinear maneuvers. Adequate control through the nonlinear maneuvers requires not only consistent open loop reference commands but linear perturbation gain-scheduled controllers as well. The gain-scheduled perturbation controllers are designed efficiently by decomposing the desired eight maneuvers into 15 linear design points for both the fixed and variable throttle cases.

##### 4.1.1 The F-15 Nonlinear Tabular Model

The nonlinear dynamics of the F-15 can be represented over the envelope with a sufficient number of trim values  $\bar{X}$  and  $\bar{U}$ : 145 points were stored, distributed as shown on the altitude-Mach plane in Figure 4-1.

A much coarser grid of flight conditions, than shown in this figure for the reference trajectory, was used for the linear perturbation models about the trajectory, and consequently in the linear control design stage. Table 4-1 shows the coarse discretization considered. Since it becomes increasingly difficult to trim the aircraft, particularly at high load factors, in the off-diagonal points in the altitude-Mach plane, only five conditions, the diagonal ones indicated in Table 4-1, were used with the three different load factors -- 1g, 3g, and 4g. The eight maneuvers were initiated at different conditions (see Section 3) to exercise controllers at various points on the flight envelope.

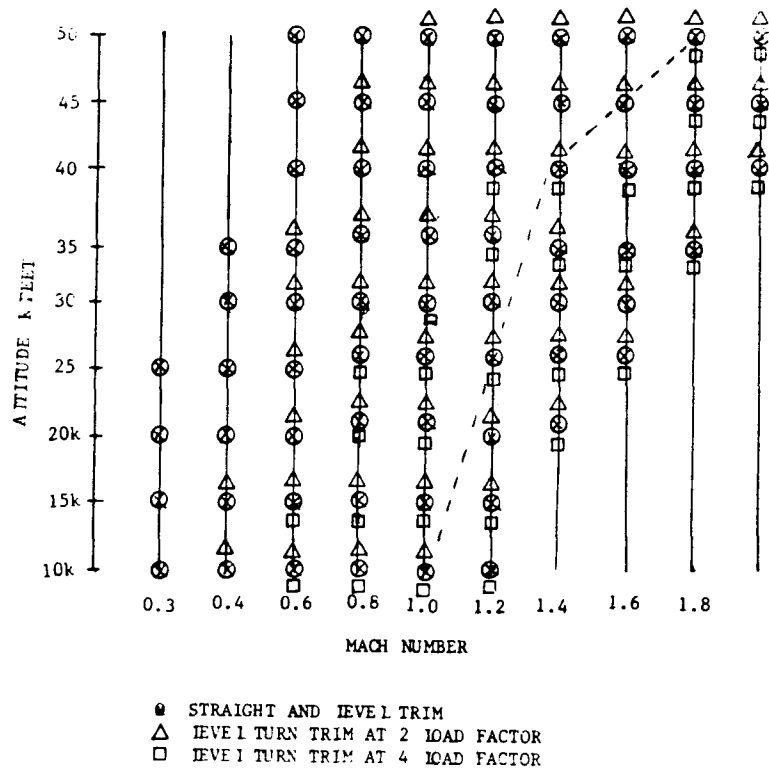


Figure 4-1. F-15 Nonlinear Tabular Model Trim Points

TABLE 4-1. F-15 LINEAR PERTURBATION MODEL AND DESIGN CONDITIONS

h/M	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
50k							X	
40k					X			
30k				X				
20k			X					
10k		X						

The trim conditions used in this work are those with  $\dot{\underline{x}} = 0$ , namely the net forces and moments are zero. Since altitude rate is zero, a load factor greater than one automatically pulls the aircraft into a level turn trim.

The static nature of the trim points is overcome in constructing a dynamic reference trajectory by assuming a ramp in velocity for a specified time of transition between two trim points. This constant  $dV/dt$  yields the thrust adjustment necessary to add to the trimmed thrust for a "dynamic" reference thrust and  $\underline{V}$  command. Conceptually, one could solve for an adjusted  $\underline{x}$  command in all the states; however, the required computation and storage is large. Therefore, only the feedforward thrust (or throttle) command is adjusted as described here, and the linear perturbation controller generates the extra transient control commands necessary for all the states to transition between the trim points. This mechanization is discussed more fully in the next subsection.

In summary, the nonlinear F-15 model required for eight nonlinear maneuvers has been represented by a tabular description of trim values of the nonlinear simulation at discrete points on the flight envelope. The number of points used to represent the reference command is much finer than for the linear models because the reference command effectively contains the nonlinear behavior in it. Load factors greater than one, level turn trims, have been used to generate the asymmetric models.

By keeping the linear perturbation model grid coarse, only a few perturbation controllers need be designed, reducing both linear model scheduling and gain scheduling requirements in the linear time-varying simulation validations. Nonlinear simulations in the next research phase will confirm whether or not the number of design points is sufficient. Accurately speaking, the "controller", is not merely the linear perturbation gains, but the way they are mechanized along with the reference commands.

#### 4.1.2 Linear Perturbation Controllers

Currently, control systems for nonlinear plants are synthesized using perturbation models or the so called linearized plant models in conjunction with the powerful linear system design approaches. The controllers so obtained may be termed Linear Perturbation Controllers to denote the linear nature of the controllers and to indicate their function, viz, controlling perturbations about the reference condition. If the system is required to track a given command, the perturbation models need to be generated at several operating points along the command history and controllers designed. In highly nonlinear systems such as aircraft, these controllers can display significant variations, often requiring these to be scheduled as a function of the independent variable.

The objective of the controller is to ensure that a given nonlinear system

$$\dot{\underline{X}} = f(\underline{X}, \underline{U}) \quad (4.1)$$

follows a given trajectory  $\underline{X}(t)$ . Here  $\underline{X} \in \mathbb{R}^n$ ,  $\underline{U} \in \mathbb{R}^m$ . To obtain the perturbation models or the linearized models, some points along the desired trajectory  $\underline{X}(t)$  are selected, say  $\underline{X}_1, \underline{X}_2, \underline{X}_3 \dots$ . A set of controls corresponding to these points,  $\underline{U}_1, \underline{U}_2, \underline{U}_3 \dots$  are next computed such that

$$f(\underline{X}, \underline{U}) = 0 \quad (4.2)$$

Note that this is not the only possible approach. If the desired trajectory satisfies  $\dot{\underline{X}} = f(\underline{X}, \underline{U})$  for nonzero  $\dot{\underline{X}}$ , then it can be used in the subsequent development.

To derive the perturbation model with state perturbations  $\delta X$  and control perturbations  $\delta U$ , let

$$X = \underline{X} + \delta X \tag{4.3}$$

$$U = \underline{U} + \delta U$$

Expanding the nonlinear system (4.1) about  $\underline{X}$ ,  $\underline{U}$  and retaining only the first order terms (this implies that the perturbation are small), one has

$$\delta \dot{X} = f_X \delta X + f_U \delta U \tag{4.4}$$

The subscripts denote partial derivative matrices. Note that  $f_X$  and  $f_U$  depend on  $\underline{X}$ ,  $\underline{U}$ . The expression (4.4) describes a linear dynamic system for which a controller of the form

$$\delta U = K \delta X \tag{4.5}$$

can be designed at the operating points  $\underline{X}_1, \underline{X}_2, \dots$

Next, to ensure that the system transits through these operating points, the following procedure is setup.

Assume that a linear interpolation scheme between  $\underline{X}_1, \underline{X}_2, \dots \underline{X}_n$  adequately describes the desired trajectory. Further, assume that at any

interpolated reference flight conditions  $\underline{X}_j$ , in between  $\underline{X}_i, \underline{X}_{i+1}$ ;

$f(\underline{X}_j, \underline{u}_j) \approx 0$ . In this case, the perturbation model (4.4) is given by

$$\delta \dot{\underline{X}} = f_{\underline{X}} \delta \underline{X} + f_{\underline{U}} \delta \underline{U} - \dot{\underline{X}}(t) \quad (4.6)$$

Note that  $\dot{\underline{X}}$  is a piecewise constant function and appears as a disturbance in the perturbation model.

As noted earlier, the perturbation controller is designed with  $\dot{\underline{X}} = 0$ . Recalling that

$$\underline{X} = \delta \underline{X} + \underline{X}$$

$$\underline{U} = \delta \underline{U} + \underline{U} \quad (4.7)$$

$$\dot{\underline{X}} = \delta \dot{\underline{X}} + \dot{\underline{X}}$$

Using (4.7) in (4.6), the linearized equations can be put in the form

$$\dot{\underline{X}} = f_{\underline{X}}(\underline{X} - \underline{X}) + f_{\underline{U}}(\underline{U} - \underline{U})$$

or

$$\dot{\underline{X}} = f_{\underline{X}} \underline{X} + f_{\underline{U}} \underline{U} - \underline{Z}(t) \quad (4.8)$$

where

$$\underline{Z}(t) = f_{\underline{X}} \underline{X} + f_{\underline{U}} \underline{U}$$

Similarly, the perturbation controller (4.5) becomes

$$\underline{U} - \underline{U} = K(\underline{X} - \underline{X})$$

or

$$\underline{U} = K(\underline{X} - \underline{X}) + \underline{U} \quad (4.9)$$

Expression (4.9) indicates the implementation of the linear perturbation controller, given in Figure 4-2. for clarity.

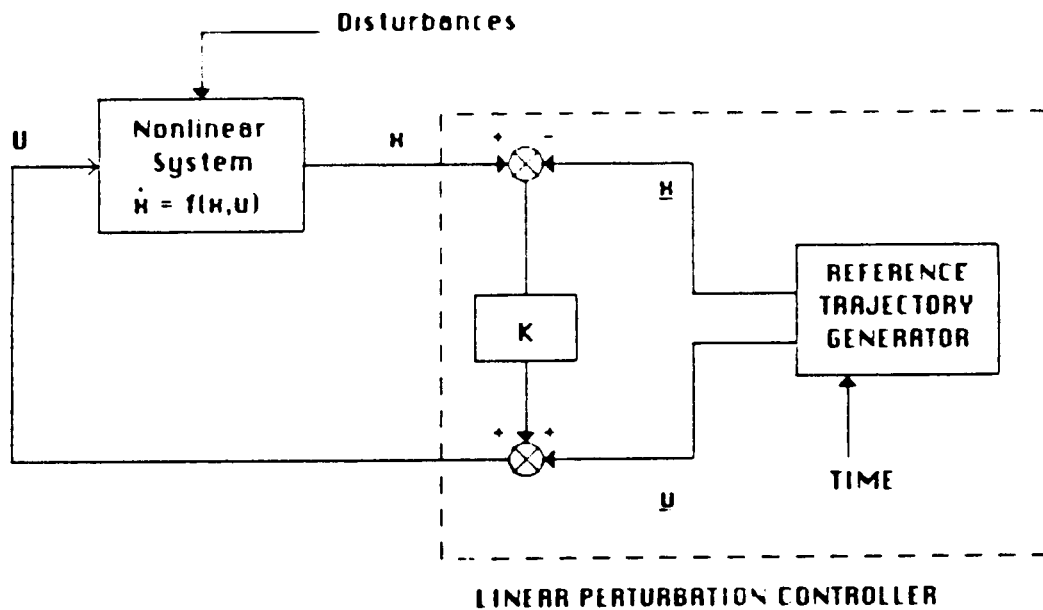


Figure 4-2. Implementation of the Linear Perturbation Controller

Note that the gain matrix  $K$  varies as a function of some scheduled variable. Expression (4.8) has an interesting interpretation, viz, when the perturbations about the desired path are small, it describes the nonlinear system dynamics with a high degree of fidelity. Hence, under closed loop control, the dynamics given by (4.8) will be very close to that of the

original nonlinear system (4.1). This fact means that linear, time varying simulations can be mechanized in a straightforward way from the relationships and models discussed here. While section five reviews the concept of linear time-varying simulation before giving the validation results, it is convenient to continue our line of thought and go over the mechanization here in the maneuver autopilot design section.

#### 4.1.3 Linear Time Varying Simulation

As noted elsewhere, for highly nonlinear systems such as aircraft, the gain matrix  $K$  in (4.9) would display large variations as a function of the flight condition. And hence, some type of scheduling strategy will be essential for the satisfactory operation of the control system. To evolve the scheduling strategy, it is desirable to have a simulation of the system which has lesser complexity than the original nonlinear system.

Examining expression (4.8) in view of the above, one finds that the partial derivative matrices  $f_x$  and  $f_u$  as functions of  $\underline{X}$ ,  $\underline{U}$  have already been computed at the controller design stage. The reference trajectory  $\underline{X}$ ,  $\underline{U}$  is also known. Since this expression describes the nonlinear system dynamics adequately for small perturbations, it may be used to develop a linear time varying simulation to evaluate the controller scheduling. Figure 4-3 gives the formal structure of the linear time varying simulation. This block diagram is structurally similar to the linear perturbation controller implementation given in Figure 4.2. The essential difference between them is that the nonlinear aircraft model has been replaced by a linear time-varying model with disturbance inputs. As mentioned in Section 4.1.2,  $\underline{X}$  and  $\underline{U}$  are linearly interpolated between time points. In order to simplify the mechanization, the partial derivative matrices  $f_x$  and  $f_u$  are stored at the same time points and linearly interpolated. Thus, along a contemplated maneuver, three or four  $\underline{X}$ ,  $\underline{U}$ , points are chosen and the corresponding  $f_x$ ,  $f_u$ , matrices are stored. Note that in the present analysis the  $\underline{X}$ ,  $\underline{U}$ , points



are chosen such that at an intermediate point  $f(\underline{X}_j, \underline{U}_j) \approx 0$ . In the course of simulations, if it turns out that in order to track the required  $\underline{X}$  history, the total control required is greater than that available, it is indicative that either the assumed maneuver time is unrealistic or that the model is inadequate or both.

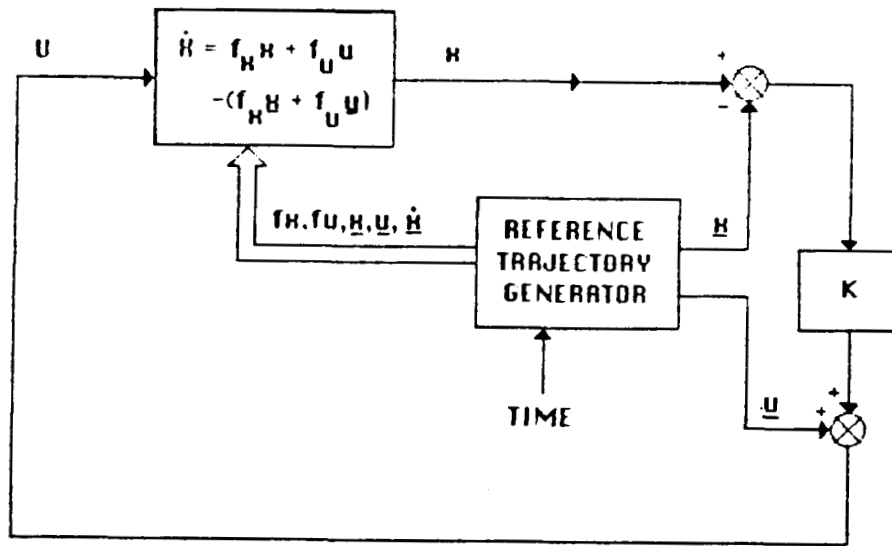


Figure 4-3. Linear Time Varying Simulation

A description of how a generic time varying simulation can be implemented in the MATRIX<sup>TM</sup> SYSTEM\_BUILD<sup>TM</sup> model building and simulation program is given in Appendix D.

## 4.2 LINEAR DESIGN TECHNIQUES

Given a small set of linear trim conditions, linear perturbation controllers can be designed to track desired reference commands. The simplest practical controllers, once the outputs are augmented with appropriate integrators for desirable steady state tracking, are output feedback controllers. Two different output feedback approaches were evaluated, viz, eigenstructure assignment and the minimum error excitation technique.

### 4.2.1 Eigenstructure Assignment Design

As described in Section 2, the model through which the desired maneuver autopilots (MAP) must work includes the fully augmented aircraft: This actually makes the control design task more difficult than dealing only with the bare airframe. The combination of CAS states and integral error states give a considerable number of eigenvalues that are slow, and therefore must be moved, and also exhibit a reasonable degree of state coupling. This coupling results in some sensitivity of results to the choice of eigenspace requested. The model used and specific results are given in Appendix A. Complete details of this eigenassignment approach can be found in [13]. It is perhaps of interest to note that in addition to the capability to handle controller structure constraints, this technique can be modified to accept partial specification of eigenvectors. This feature can be valuable in high order systems. In summary, to employ this synthesis approach, the following are required.

- (i) Based on the practical aspects of the problem, choose a minimal set of measurements which will permit the designer to achieve the desired performance. Introduce dynamic compensators such as integral feedbacks, lead-lag networks, etc., based on experience.
- (ii) Choose a set of desired eigenvalues and eigenvectors equal to the number of outputs.

While the selection of desired eigenvalues is often apparent in a given problem, the desired eigenvectors are difficult to select. Three approaches were developed to help guide this choice, viz, minimally restructured eigenassignment, decoupling eigenassignment and dominant mode eigenassignment.

#### 4.2.1.1 Minimally Restructured Eigenassignment

Since it is known that the closed loop eigenvectors lie in a subspace spanned by the columns of  $(\lambda_i I - A) B^{-1}$ ,  $i = 1, \dots, n$ , linear combinations of these vectors were used as desired eigenvectors. The weights to be used in generating these linear combinations were constructed from the additional information that unassigned eigenvectors should be close to their open loop values in a least square sense.

#### 4.2.1.2 Decoupling Eigenassignment

Since we are interested in having the least cross axis coupling in the controller as possible, the desired eigenvectors in the longitudinal channel may be chosen so that the responses from lateral channels are blocked, i.e., select eigenvectors as

$$\begin{bmatrix} (\lambda_i I - A) & B \\ C_{LAT} & 0 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_d = \sum_{i=1}^n \alpha_i v_i$$

$V_d$  are the desired eigenvectors.  $\alpha_i$  are selected using the same criteria as in the minimally structure case. Though these two approaches could be made to work at each flight condition by iterating on the desired eigenvalues, they failed to easily produce a set of acceptable eigenvectors which could be used at other flight conditions. Next, partial specification of desired eigenvectors was attempted using the dominant mode approach.

#### 4.2.1.3. Dominant Mode Eigenassignment

According to ref [13], complete specification of desired eigenvectors are neither necessary nor desirable. Depending on the states that should or should not participate in a given mode, appropriate entries in the eigenvectors are made ones or zeros, leaving other entries free. With these eigenvectors, the desired eigenvalues are moved as far left from the imaginary axis as possible with least change in the location of unplaced eigenvalues. The desired eigenvectors so obtained appeared to work over most flight conditions. Note, however, that extensive iterations on the desired eigenvalues may often be required to produce a satisfactory design.

#### 4.2.1.4 Conclusions on Output Eigenstructure Assignment

Specific maneuver autopilot design results are discussed in Appendix A, along with the more complete description and evaluation of output eigenstructure assignment for application to flight test trajectory control, including difficulties in selecting desirable eigenvalues and eigenvectors. This approach demands several iterations to converge to a satisfactory design and does not appear to easily give suitable insight for output feedback design of high order multivariable systems which will be used at other operating points. If a rational method to generate an achievable set of eigenvectors is devised, this technique will be made more attractive. One possibility might be to generate gradients of the eigensystem between flight conditions and include this information in the single point design technique. The next two subsections describe two other techniques of output feedback gain solution.

#### 4.2.2 Minimum Error Excitation Output Feedback Design

Following Kosut's [8] notation in describing his development of the minimum error excitation output feedback design method, for the linear system

$$\dot{x} = Ax + Bu,$$

$$y = Hx,$$

one first designs a full state LQ regulator

$$J = \frac{1}{2} \int_0^{\infty} (x^T H^T Q_y H x + u^T R u) dt,$$

with the optimal feedback control law  $u^* = F^*x$ .

Both because algebraic output feedback methods do not guarantee stability and because of the presence of slow closed loop full-state feedback modes (either neutrally stable unobservable modes or other sluggish phugoid like, spiral or integral error modes), it is desirable to design with a guaranteed stability margin [14, 15]. Specifying a stability margin of  $\alpha$  ensures that the real parts of all closed loop roots are less than  $-\alpha$ . This is equivalent to optimizing the performance index

$$J = \frac{1}{2} \int_0^{\infty} e^{-2\alpha t} (x^T H^T Q_y H x + u^T R u) dt,$$

and can be accomplished by destabilizing the open loop plant by  $\bar{A} = A + \alpha I$ ; solving for the corresponding optimal gains in the standard LQ problem, and using them with the original open loop dynamics matrix  $A$ . Costs were chosen according to Bryson's rule [15], the inverse of the squared deviation desired on inputs and outputs, with a scalar factor between  $Q$  and  $R$  to regulate the extent of high gain solution achieved.

A minimum norm output error feedback law can be determined from

$$u = C_y y, \quad \text{with}$$

$$C_y = H^+ F^*,$$

where  $H^+$  is the pseudoinverse of  $H$ . This is merely a least squares projection of the full state gains onto the output subspace.

The sensitivity of the projection described above can be minimized by doing a weighted least squares, where the weight is the closed loop state covariance excited by a unit error density. One merely solves the Lyapunov equation

$$(A + BF^*)P + p(A + BF^*)^T + I = 0,$$

to obtain the constrained gain

$$F_y = F^* P H^T (H P H^T)^{-1} H, \text{ or}$$

$$C_y = F^* P H^T (H P H^T)^{-1}$$

The destabilized open loop plant  $\bar{A}$  was used instead of  $A$  in the Lyapunov equation solution to retain as much of the full-state law guaranteed stability margin as possible.

Maneuver autopilots were designed at all flight conditions within two iterations using the above output feedback design procedure. The design criterion were

$\text{Re}(\lambda_1) < -.2 \Rightarrow$  a 5 second maximum time constant in tracking, and

$\xi > .7 \Rightarrow$  low overshoot.

With the integral error states and general low frequency oscillatory poles pairs due to lateral/longitudinal coupling, the plant is not a simple one to control. Since Mach and load factor were chosen as the scheduling variables, 15 designs were generated - five designs at three load factors shown in Table 4-2. The state and control weighting matrices used in this synthesis is listed in Appendix E. The corresponding output feedback gains are also given in this appendix.

Since the zoom-and-push-over and excess thrust windup turn require fixed throttle, 15 more designs were generated without this control, also shown in Table 4-2.

Typical initial condition responses in a straight and level flight condition at 40K feet and Mach of 1.4 are shown in Figure 4-4.

TABLE 4-2. SLOWEST MODE BEHAVIOR AT ALL DESIGN CONDITIONS

(Subscript 1 corresponds to the design with all controls, subscript 2 corresponds to the design without throttle control)

h, m						
LOAD	h = 10000'	h = 20000'	h = 30000'	h = 40000'	h = 50000'	
FACTOR	M = 0.8	M = 1.0	M = 1.2	M = 1.4	M = 1.8	
1	$\lambda_1 = -0.277$ $\xi_1 = 0.996$	$\lambda_1 = -0.289$ $\xi_1 = 0.97$	$\lambda_1 = -0.3$ $\xi_1 = 0.95$	$\lambda_1 = -0.308$ $\xi_1 = 0.945$	$\lambda_1 = -0.292$ $\xi_1 = 0.94$	
	$\lambda_2 = -0.28$ $\xi_2 = 1$	$\lambda_2 = -0.257$ $\xi_2 = 1$	$\lambda_2 = -0.234$ $\xi_2 = 1$	$\lambda_2 = -0.2276$ $\xi_2 = 1$	$\lambda_2 = -0.219$ $\xi_2 = 1$	
2	$\lambda_1 = -0.28$ $\xi_1 = 0.99$	$\lambda_1 = -0.26$ $\xi_1 = 0.94$	$\lambda_1 = -0.23$ $\xi_1 = 0.9$	$\lambda_1 = -0.2274$ $\xi_1 = 0.2$	$\lambda_1 = -0.225$ $\xi_1 = -0.855$	
	$\lambda_2 = -0.288$ $\xi_2 = 0.99$	$\lambda_2 = -0.268$ $\xi_2 = 0.947$	$\lambda_2 = -0.254$ $\xi_2 = 0.947$	$\lambda_2 = -0.276$ $\xi_2 = 0.958$	$\lambda_2 = -0.242$ $\xi_2 = 0.88$	
4	$\lambda_1 = -0.267$ $\xi_1 = 0.99$	$\lambda_1 = -0.23$ $\xi_1 = 0.92$	$\lambda_1 = -0.209$ $\xi_1 = 0.866$	$\lambda_1 = 0.17$ $\xi_1 = 0.98$	$\lambda_1 = -0.116$ $\xi_1 = 0.95$	
	$\lambda_2 = -0.28$ $\xi_2 = 0.99$	$\lambda_2 = -0.23$ $\xi_2 = 1$	$\lambda_2 = 0.243$ $\xi_2 = 0.95$	$\lambda_2 = -0.2405$ $\xi_2 = 1$	$\lambda_2 = -0.209$ $\xi_2 = 0.895$	



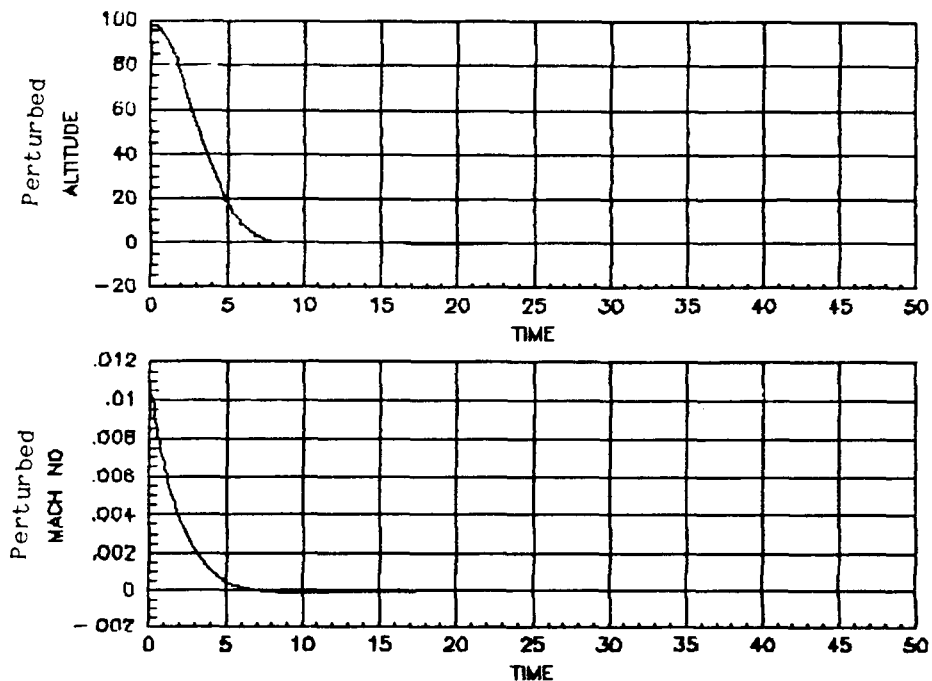


Figure 4-4. Initial Condition Response for the Output Feedback Minimum Error Excitation Perturbation Controller (Reference Condition  $H = 40k'$ ,  $Mach = 1.4$ , Straight and Level Flight)

Section 5 gives the tracking behavior of the output feedback minimum norm controllers in the eight maneuvers of interest.

#### 4.3 NONLINEAR FLIGHT TEST TRAJECTORY CONTROLLERS

Research on nonlinear flight test trajectory control (FTTC) design was conducted with the goal of completely eliminating the need for gain scheduling. A brief literature review is given below, outlining how recent theoretical work can be applied to the FTTC problem before a demonstration problem is given to illustrate the approach.

This research deals with the synthesis of nonlinear flight test trajectory controllers using the recent results in prelinearizing transforms due to Meyer [17-33] and singular perturbation theory [34-36]. The use of singular perturbation theory in this problem simplifies the command generation scheme in addition to providing a consistent approach for eliminating ignorable state variables. The prelinearizing transformations are more transparent in this formulation. The slow-fast computations are clearly separated and can be carried out at different rates on the flight control computer. It is interesting to note that in Ref. 21, even though the controller development did not make use of singular perturbation theory, the time-scale separation formed a basis for implementation on the flight control computer. A schematic block diagram of the slow-fast flight test trajectory controller is given in Figure 4-5.

Flight test controller synthesis will be developed for the F-15 fighter aircraft in the next contract phase. For the F-15 and most fighter aircraft, it can be assumed that the aircraft under consideration has the four usual controls: throttle, aileron, rudder and elevator. The objective of the flight test controller is to track the given commands in airspeed, angle of attack, angle of sideslip and altitude in presence of disturbances and modeling imperfections. It is clear that the commanded trajectory has to be executable by the aircraft under consideration. Note that the flight test control problem discussed here is distinct from those described by Meyer, et. al. [20-22] since, in their work, the trajectory to be followed consisted of the three position components specified as functions of time.

Modeling and time-scale separation can be exploited to a considerable degree in this approach. The mechanization details of prelinearization and the slow-fast controller synthesis for general flight test maneuvers will be given in the next project phase based on the problem formulation developed here.

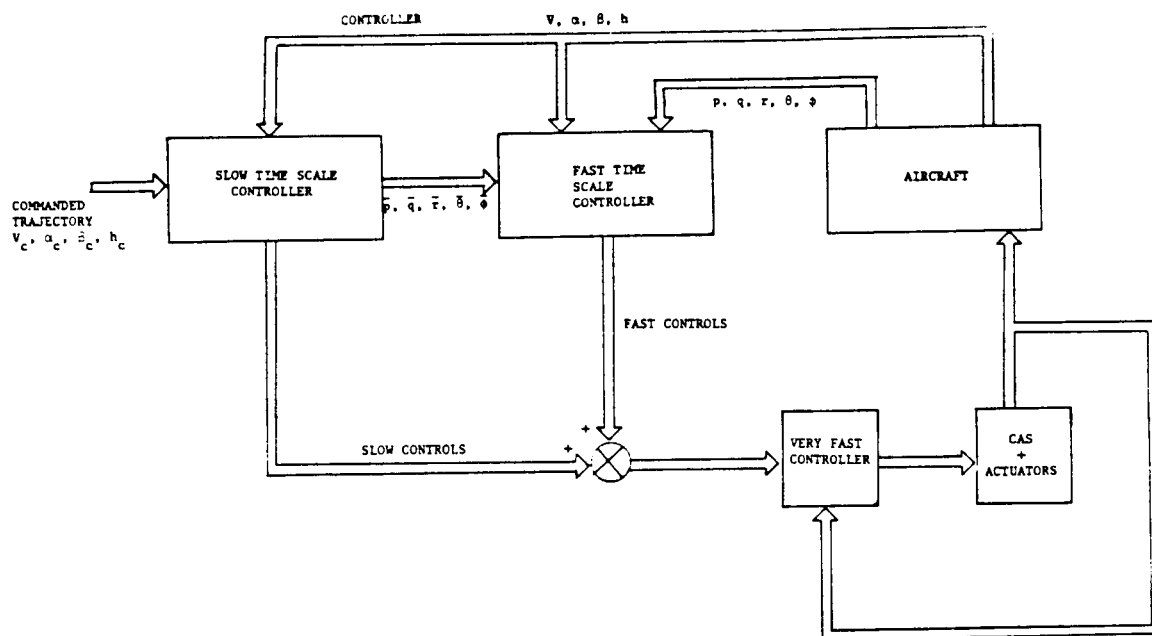


Figure 4-5. Singular Perturbation Nonlinear Flight Test Trajectory Controller.

Slow states:  $V, \alpha, \beta, h$

Fast states:  $p, q, r, \theta, \phi$

Very fast states: CAS and Actuator states

An illustrative example of this nonlinear controller approach is given in Appendix B.

## SECTION 5

### SIMULATION AND EVALUATION

The performance of perturbation controllers for initial condition errors was verified during the design phase. Some techniques for evaluating their tracking performance will be discussed in this section. Though it is clear that the actual performance of these controllers can only be assessed with the full nonlinear aircraft model, it is desirable to introduce an intermediate validation phase to ensure a smooth transition from perturbation controller design to full nonlinear simulation. It should be emphasized at the outset that the simulations discussed here are approximate and consequently the tracking performance will differ in the nonlinear simulation validation in the next study phase.

#### 5.1 MANEUVER SIMULATION

Originally, it was decided to carry out a linear time varying simulation of the systems using linearized aircraft models along the desired flight-test trajectory as discussed in Appendix D. For a two state variable model analyzed in Appendix B, the computing time was small. However, the aircraft and CAS system has 31 states and continuous interpolation was found to be extremely time consuming. In view of the excessive computational effort required, and the limited value of the information obtained, it was then decided to switch models and controller gains along a desired flight test trajectory. This approach introduced artificial gain switching transients and also led to misleading conclusions. Hence whenever feasible, the simulations discussed here used one interpolated model, and gains based on the flight condition halfway through the maneuver. With this approach, the modeling inaccuracies will be almost equally distributed throughout the maneuver.

It should be emphasized that the simulations with linearized models will test only the feedback controller portion of the maneuver autopilot. The open loop control histories generated from maneuver modeling can be tested only during the full nonlinear simulation of aircraft and CAS.

## 5.2 MANEUVER SIMULATION MECHANIZATION

The linear perturbation equations used for design have been discussed in Section 4. In this section, these will be modified to generate linear simulations. The linearized aircraft with CAS is of the form

$$\dot{\delta x} = F\delta x + G\delta u$$

$$\delta y = H\delta x$$

The output feedback perturbation controller is of the form

$$\delta u = C_y \delta y.$$

Expanding the perturbation equations back out gives

$$\dot{\underline{x}} - \underline{x} = F(\underline{x} - \underline{x}) - GC_y H(\underline{x} - \underline{x})$$

$$\dot{\underline{x}} = (F - GC_y H)\underline{x} - [(F - GC_y H)\underline{x} - \dot{\underline{x}}],$$

from which the maneuver simulations can be mechanized. Note that  $\dot{\underline{x}}$  can be viewed as a disturbance (and neglected) or as a part of the external reference command. For the high speed maneuvers simulated here  $\dot{\underline{x}}$  clearly cannot be ignored, but were computed numerically with a forward differencing of  $\underline{x}$ .

### 5.3 MANEUVER SIMULATION RESULTS

As discussed earlier,  $\dot{\underline{x}}$  was computed numerically from  $\underline{x}$  and used as part of the reference command. Since  $\underline{x}$  has corners, there are jumps, or spurious step inputs due to the discontinuities in  $\dot{\underline{x}}$ . Using a smooth  $\underline{x}$  from quadratic or cubic spline fits to the trim points  $\underline{x}_i$  will remedy this problem; however, the effects of the discontinuities can clearly be seen in the tracking trajectories in this subsection.

In the following, typical simulation results for each flight test trajectory will be presented. The flight conditions in these simulations are chosen so that the maneuver autopilot is exercised over nearly the entire aircraft envelope. Except where indicated, in the plots that follow, the dotted lines denote the commanded variables generated from the maneuver modeling program while the solid line represents the trajectory evolution from the linear simulation.

#### 5.3.1 Transient Trajectory

In this simulation, a transient trajectory is setup to transfer the aircraft from straight and level flight conditions at 20000' altitude and Mach 0.8 to straight and level flight conditions at 30000' altitude and Mach 1.2 in 60 seconds. The simulation results are presented in Figs. 5-1 through 5-3. The altitude tracking is very good. However, the throttle history in Fig. 5-3 indicates that during the first five seconds, the controller commanded a negative throttle. This is caused by the nonminimum phase behavior of the aircraft, clearly discernable in the Mach number and

angle of attack histories. In the actual implementation, the commanded altitude can be modified to include an initial descend leg in order to compensate for the nonminimum phase behavior. Alternately, the commanded Mach number can be modified to have an initial decreasing segment. In any case, mere command modification would fix the initial negative throttle difficulty.

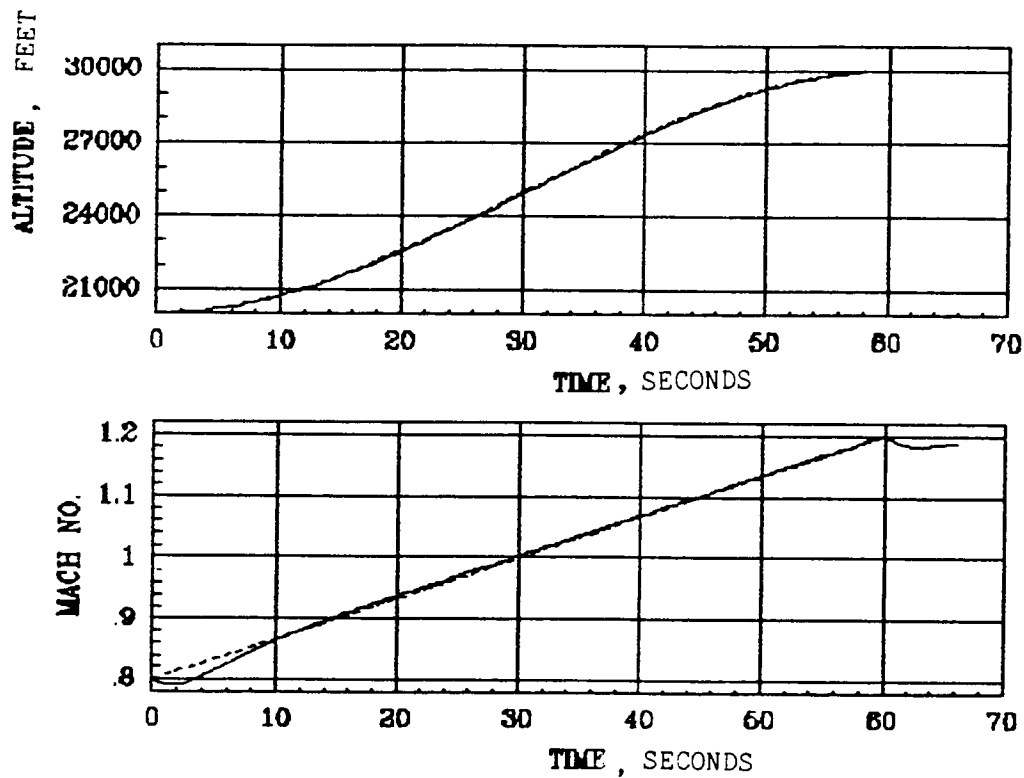


Figure 5-1. Altitude and Mach Number Evolution Along the Transient Trajectory

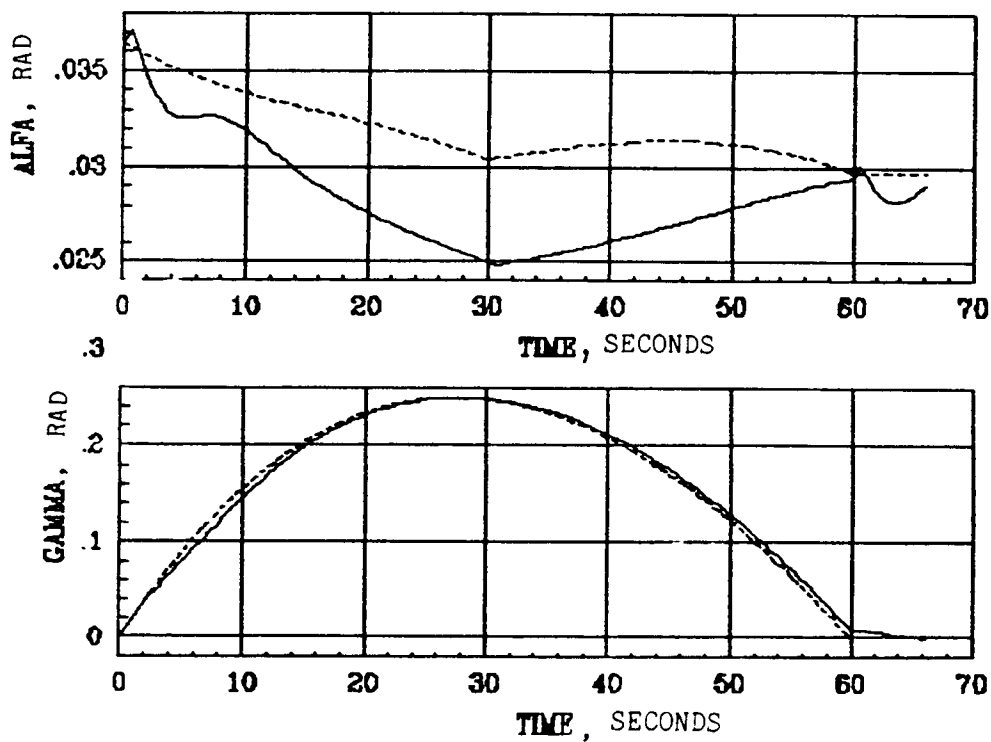


Figure 5-2. Angle of Attack and Flight Path Angle Evolution Along the Transient Trajectory



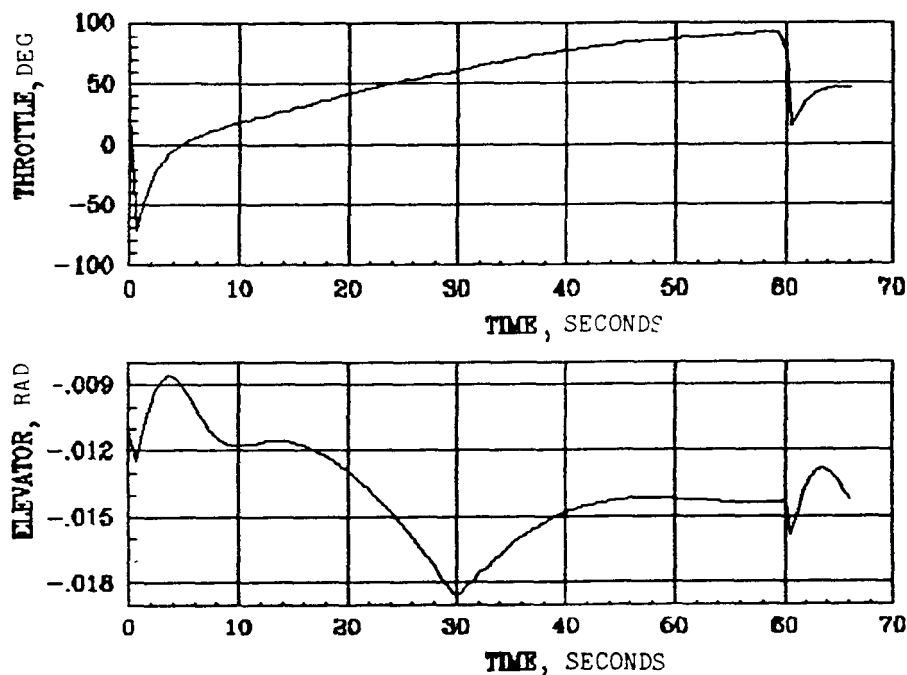


Figure 5-3. Throttle and Elevator Deflection Along the Transient Trajectory.

### 5.3.2 Level Acceleration

A level acceleration flight test trajectory at 30000' altitude is considered here. The aircraft is required to accelerate from Mach 0.9 to Mach 1.2 in 60 seconds. The tolerance on altitude is  $\pm 50'$  while Mach number error should be within  $\pm 0.01$ . The simulation results for this maneuver are presented in Figs. 5-4 through 5-6. The Controller was able to maintain the altitude within  $\pm 0.1$  feet while tracking the Mach number within the given specifications. The throttle and elevator deflections given in Fig. 5-6 are well within the saturation levels. From the maximum throttle requirement in this maneuver, it appears that the maneuver time could be decreased by about 20 seconds.

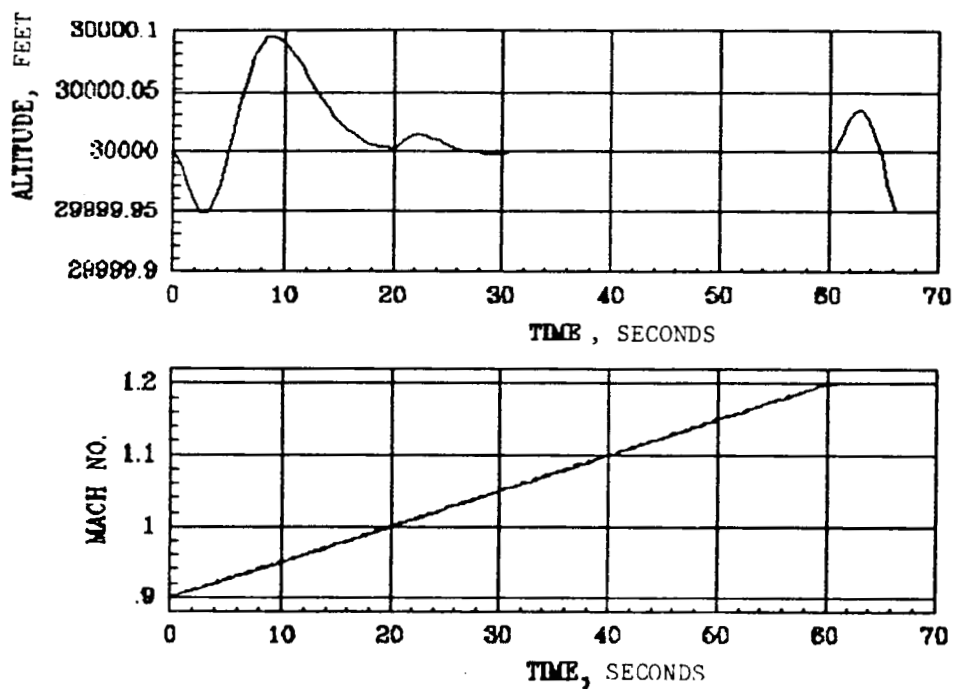


Figure 5-4. Altitude and Mach Number Evolution Along the Level Acceleration Flight Test Trajectory.

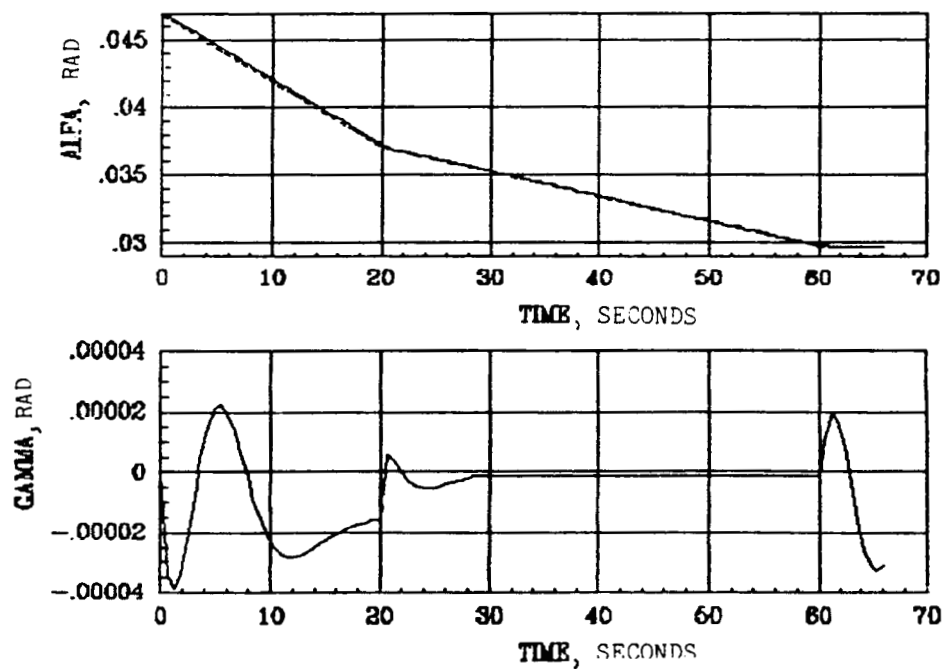


Figure 5-5. Angle of Attack and Flight Path Angle Evolution Along the Level Acceleration Flight Test Trajectory.

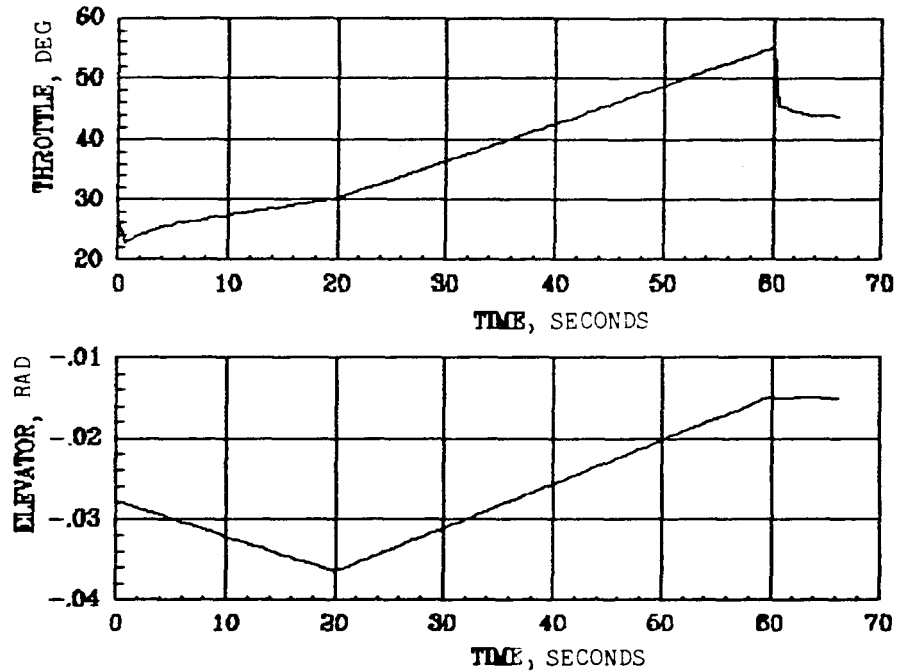


Figure 5-6. Throttle and Elevator Deflection Along the Level Acceleration Flight Test Trajectory.

### 5.3.3 Pushover, Pullup

This maneuver was initiated at 30000' altitude and 0.8 Mach straight and level flight condition. The objective is to track a pieewise linear angle of attack history given in Fig. 5-8 while maintaining the Mach number constant at the initial value. The results of the maneuver simulation are given in Figs. 5-7 through 5-9. From Fig. 5-7, it can be seen that the Mach number error is within 0.001 of the commanded value. The angle of attack tracking error is less than  $0.2^\circ$  throughout the maneuver. The throttle history given in Fig. 5-9 shows a small negative region at the imitation of pullup at 30 seconds and is primarily due to the corner in the angle of attack command. Smoothing this corner in the final mechanization would eliminate this negative throttle region.

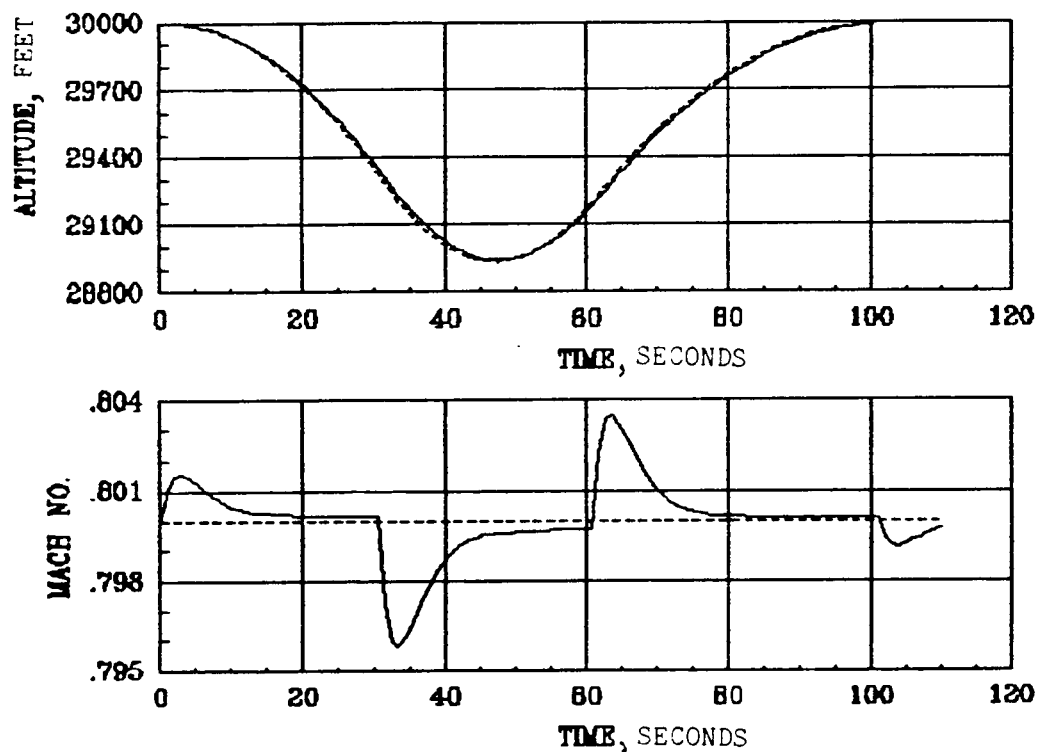


Figure 5-7. Altitude and Mach Number Evolution Along the Pushover/Pullup Flight Test Trajectory

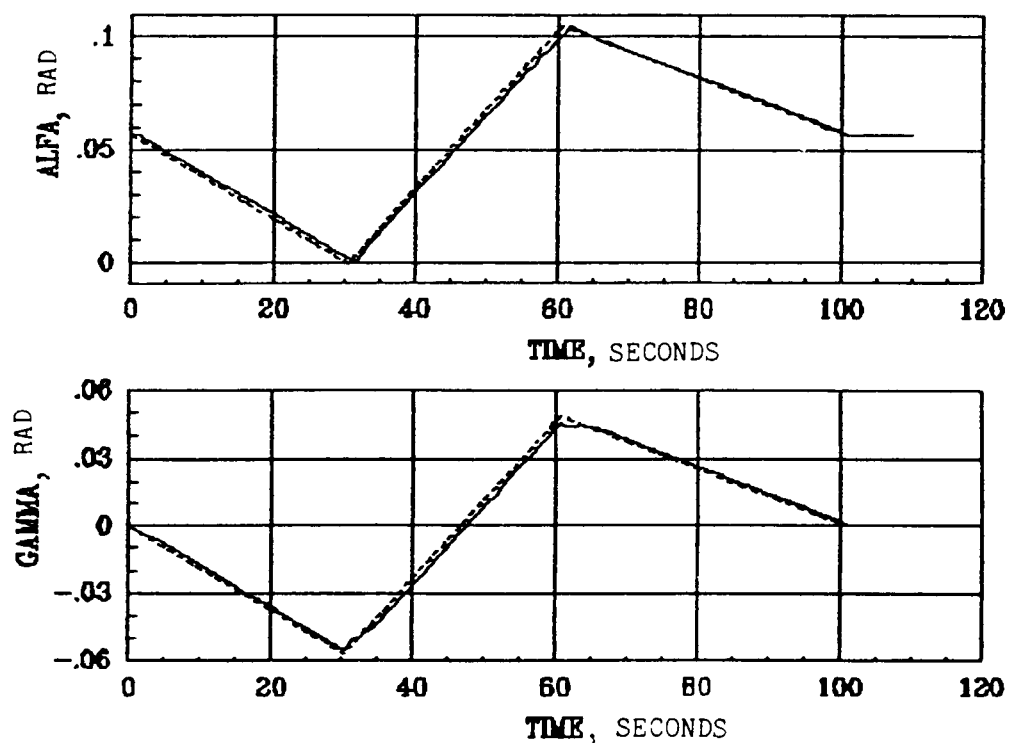


Figure 5-8. Angle of Attack and Flight Path Angle Evolution along the Pushover/Pullup Flight Test Trajectory

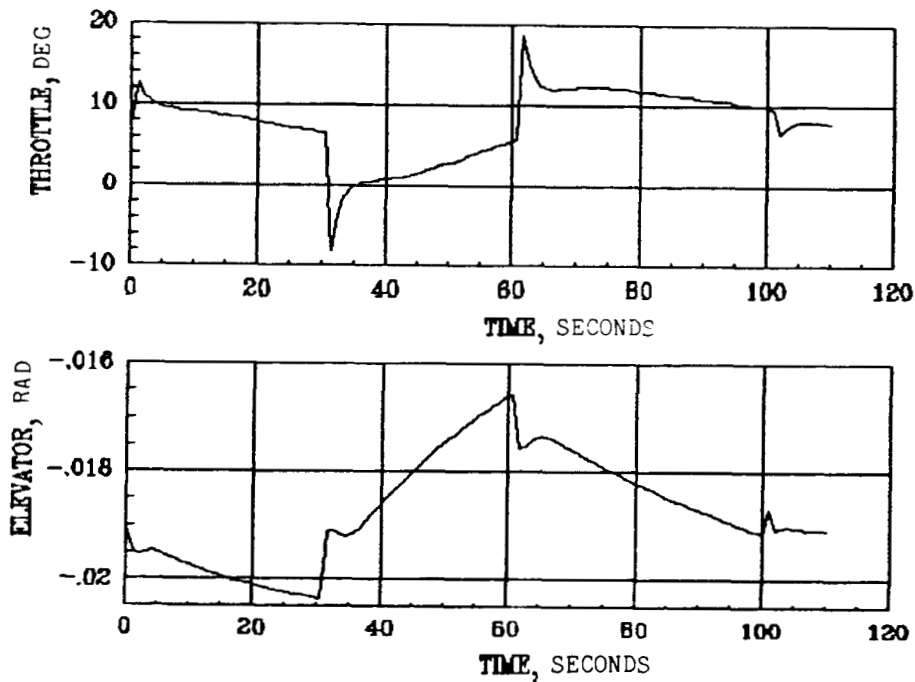


Figure 5-9. Throttle and Elevator Deflection along the Pushover/Pullup Flight Test Trajectory.

#### 5.3.4 Zoom and Pushover

As noted in the maneuver modeling, this flight test trajectory is executed in three phases. In the first phase the aircraft is transferred from straight and level flight conditions to the beginning of the zoom and pushover parabolic trajectory. The second phase consists of the zoom and pushover trajectory while the third phase restores the aircraft to the original straight and level flight condition. During the first and third phases all the controls are active, but the throttle is fixed during the zoom and pushover trajectory.

In the present case, the aircraft is first trimmed to fly straight and level at 30000' altitude and Mach 0.4. An initial transient trajectory is then executed with all controls active until the point marked A in Fig. 5-10. At this point, the throttle is fixed and the aircraft executes the parabolic zoom and pushover trajectory until the point B in this figure. The throttle is released at point B and the aircraft performs another transient maneuver to restore it to the original conditions.

Since the controller performance along the transient trajectory has already been investigated in Section 5.3.1, the tracking performance along the zoom and pushover trajectory will only be demonstrated here. The aircraft begins the zoom and pushover maneuver at about 28000' and Mach 0.45 and completes the maneuver at about the same conditions. The controller performance is illustrated in Figs. 5-10 through 5-12. The conditions at the apex of the parabola is of particular interest in this maneuver. From Fig 5-11, it can be observed that the angle of attack at the apex is within 0.005 radian of the required value. The altitude error is within 50' at the apex and the Mach number is within 0.05 of the required value. Initial transients in altitude and Mach number can be seen at point A in Fig. 5-10. These are essentially due to the availability of just one control variable, the elevator, to track three state variables: altitude, Mach number and angle of attack. Thus, an initial condition error on altitude would propagate to Mach number and angle of attack channels and vice versa.

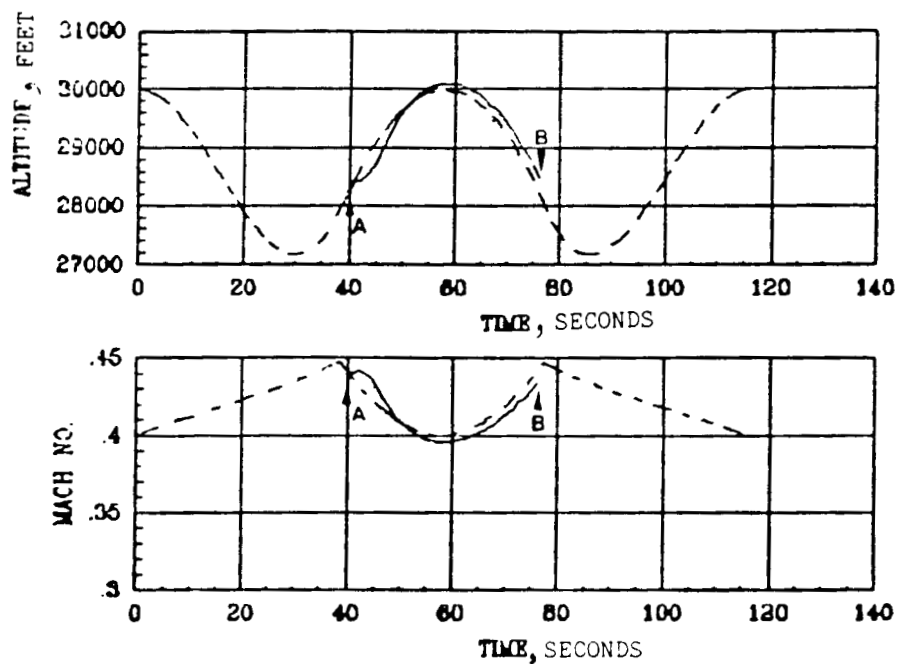


Figure 5-10. Altitude and Mach Number Evolution along the Zoom and Pushover Flight Test Trajectory.  
 A: Throttle Fixed  
 B: Throttle Released

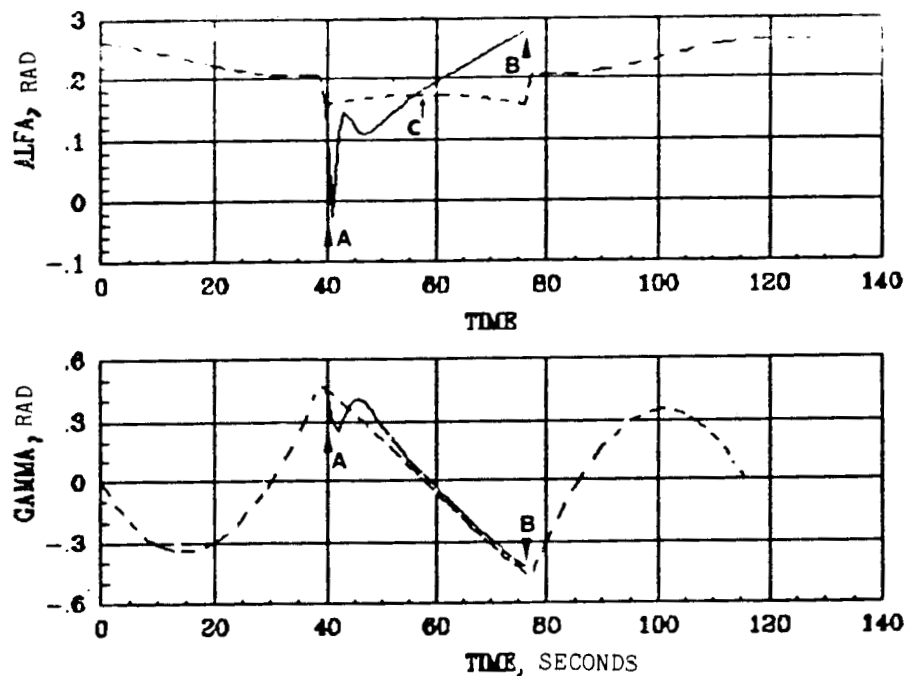


Fig. - 5-11. Angle of Attack and Flight Path Angle along the Zoom and Pushover Flight Test Trajectory.  
 A: Throttle Fixed  
 B: Throttle Released  
 C: Apex of the Parabola

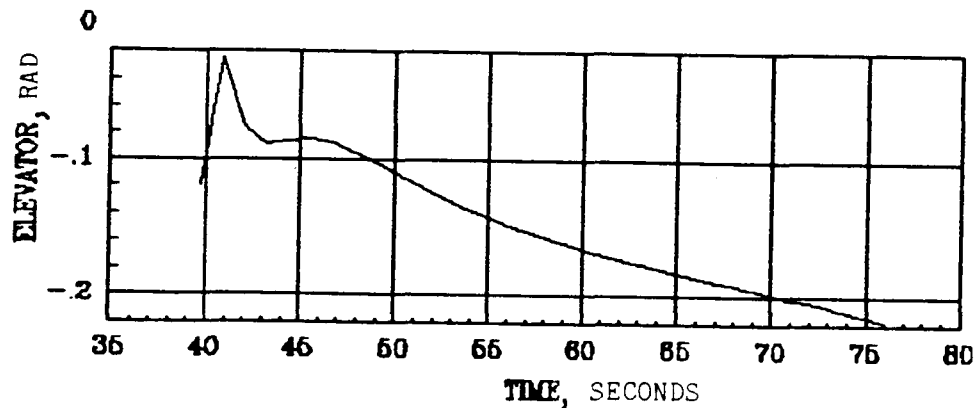


Figure 5-12. Elevator Deflection along the Zoom and Pushover Flight Test Trajectory.

#### 5.3.5 Excess Thrust Windup turn

The results for an excess thrust windup turn trajectory at 40000' altitude and Mach 1.4 are given in Figs. 5-13 through 5-19. The altitude and Mach number were required to be constant throughout the windup turn trajectory, while tracking an angle of attack command as shown in Fig. 5-14. The aircraft roll attitude in this maneuver is close to  $70^\circ$  and results in a highly coupled model to be controlled by the maneuver autopilot. It can be observed that the maneuver autopilot maintained the altitude within  $\pm 5'$  and the Mach number error is within  $\pm 0.002$ . Except at the beginning and the end of the maneuver, the angle of attack error is within 0.005 radian of the commanded value. For about 10 seconds during the beginning and end of the maneuver, the rudder deflection is close to  $12^\circ$  while in the high roll attitude region, an elevator deflection of nearly  $17^\circ$  was demanded. This indicates that at the present flight conditions, a less stringent maneuver should be attempted.



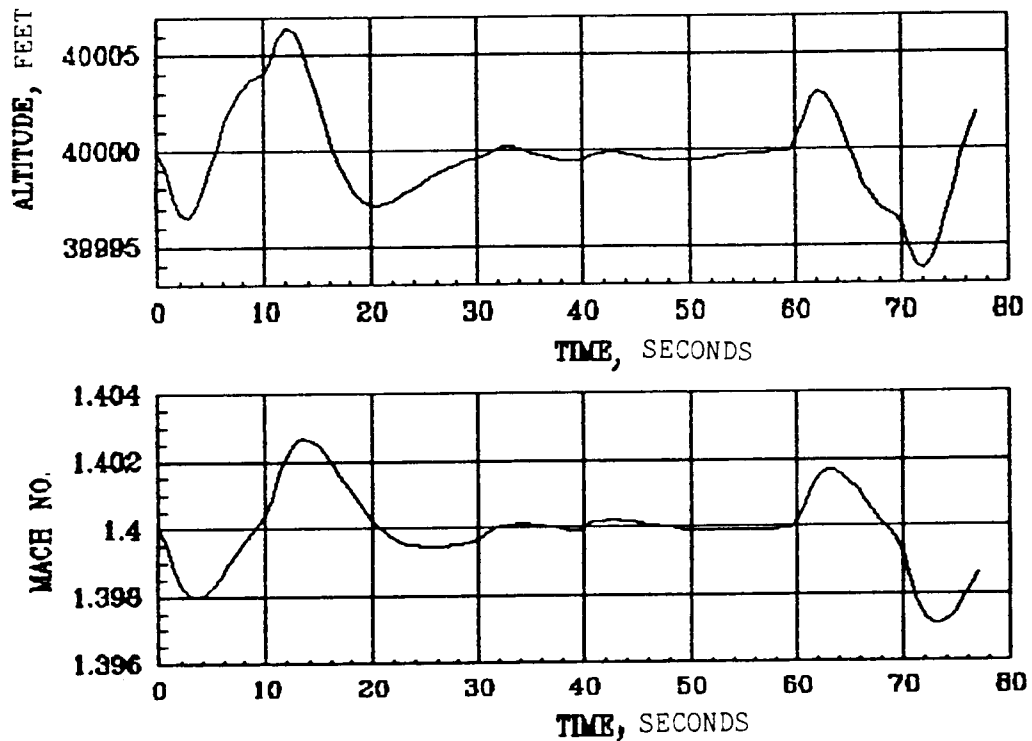


Figure 5-13. Altitude and Mach Number Evolution along the Excess Thrust Windup Turn Flight Test Trajectory

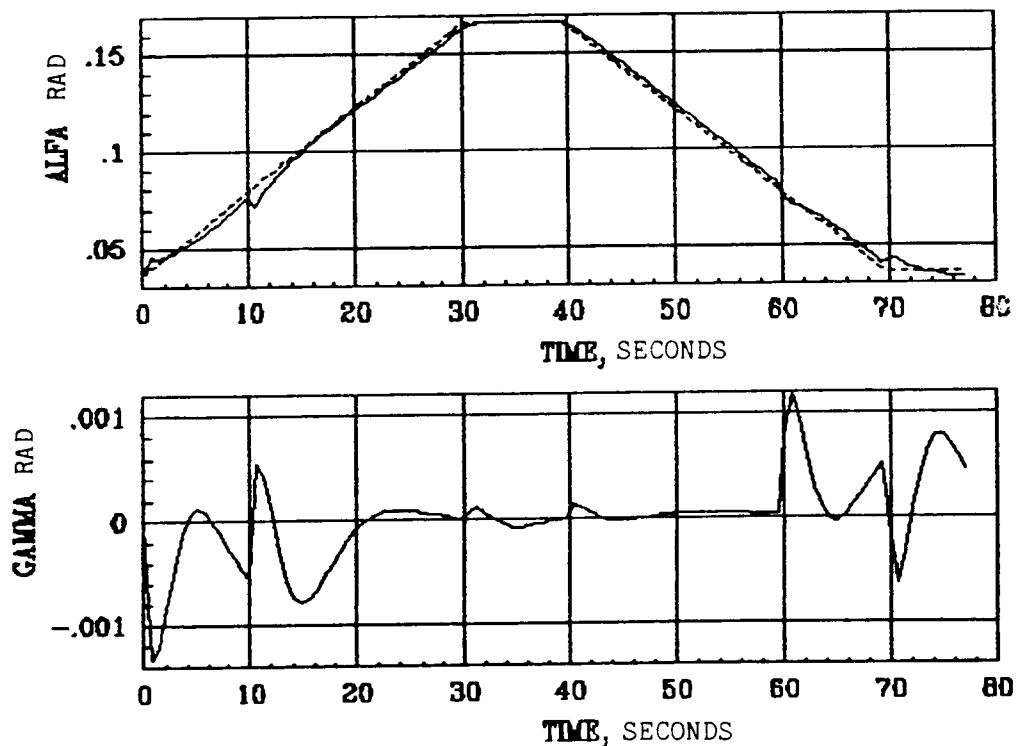


Figure 5-14. Angle of Attack and Flight Path Angle Evolution along the Excess Thrust Windup Turn Flight Test Trajectory

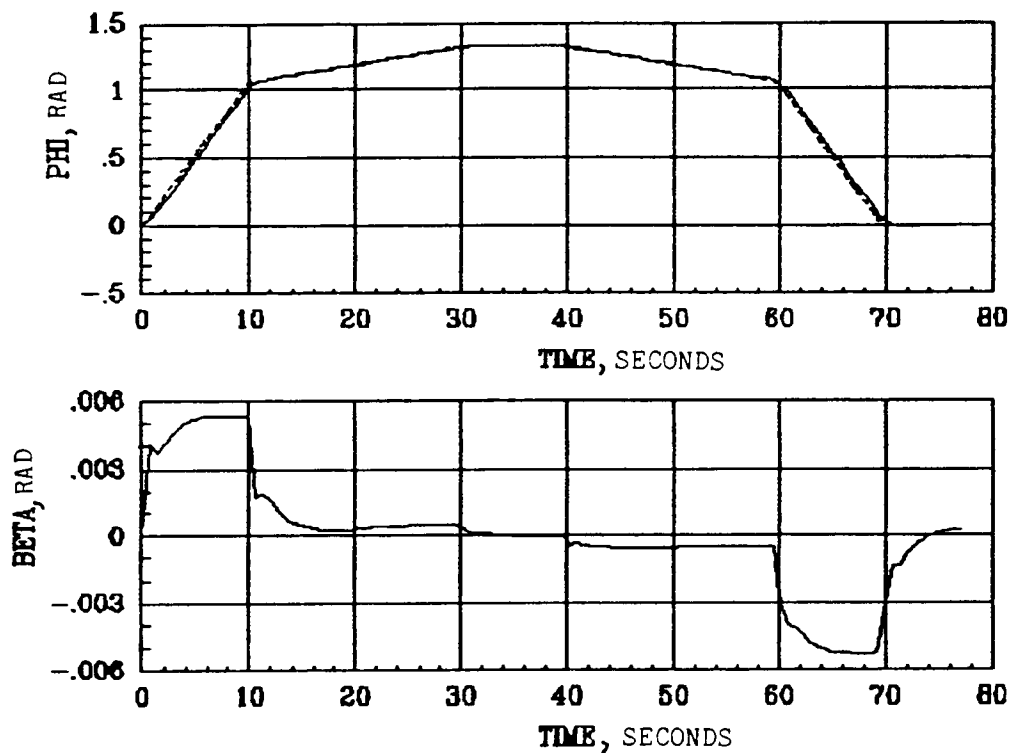


Figure 5-15. Roll Attitude and Angle of Side Slip Along the Excess Thrust Windup Turn Flight Test Trajectory

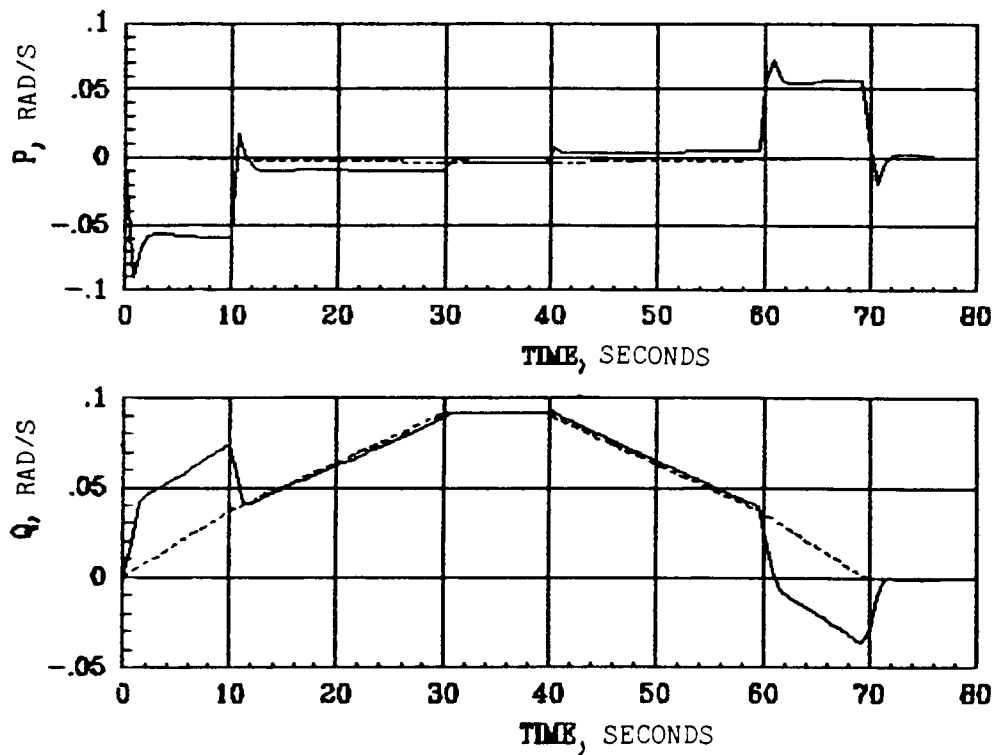


Figure 5-16. Roll and Pitch Body Rate Evolution Along the Excess Thrust Windup Turn Flight Test Trajectory

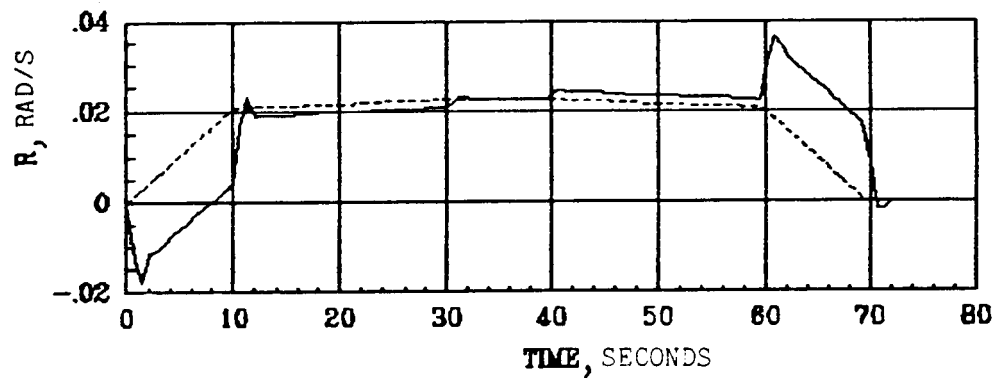


Figure 5-17. Yaw Body Rate Evolution along the Excess Thrust Windup Turn Flight Test Trajectory

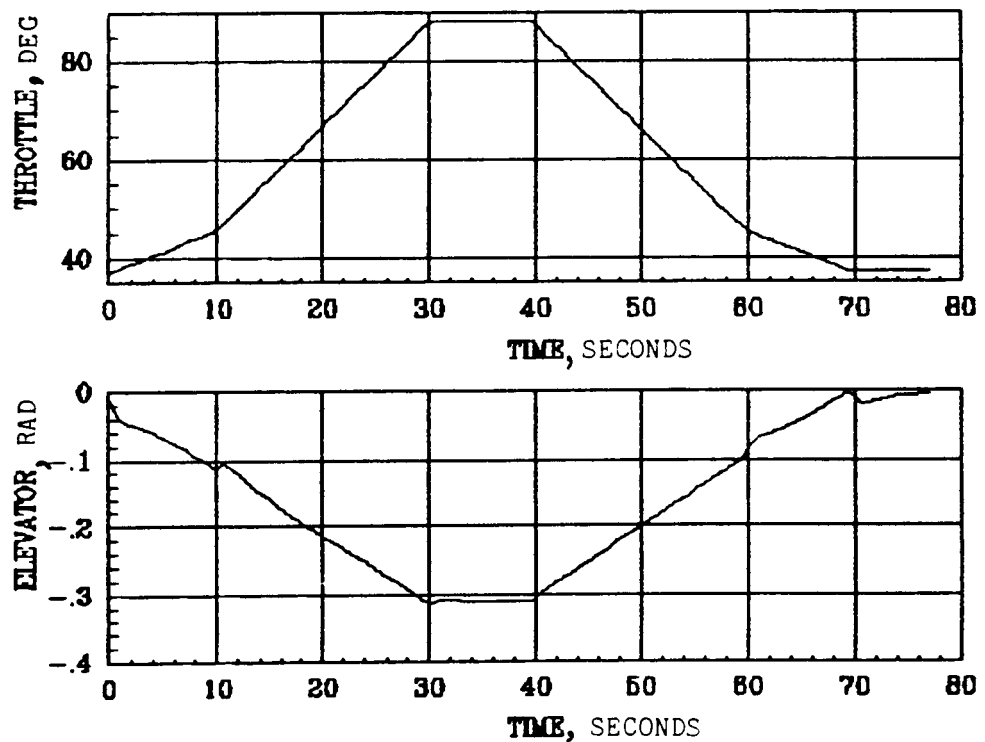


Figure 5-18. Throttle and Elevator Deflection along the Excess Thrust Windup Turn Flight Test Trajectory

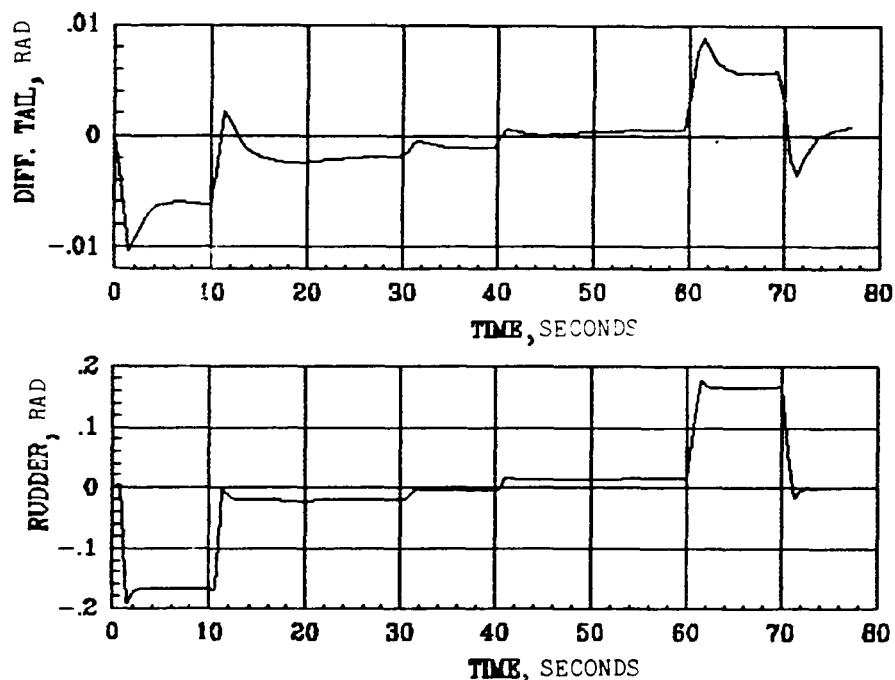


Figure 5-19. Differential Tail and Rudder Deflection along the Excess Thrust Windup Turn Flight Test Trajectory.

#### 5.3.6 Constant thrust windup turn

A descending constant thrust windup trajectory will be illustrated in the following. The aircraft starting at straight and level flight condition enters a level turn with linearly increasing angle of attack, upto 30 seconds in Fig. 5-21. At this point, the throttle is fixed and the constant thrust windup trajectory begins. At the end of the maneuver, the Angle of attack is gradually decreased to the straight and level trim values. During the constant throttle windup turn trajectory, the Mach number is to remain constant. The simulation results for this maneuver are given in Figs. 5-20 through 5-26. From Fig. 5-20, it can be seen that the Mach number was maintained within  $\pm 0.0075$  while the angle of attack tracking error was within 0.01 radians. The control surface deflections were within the saturation limits except at the point where the constant throttle windup trajectory began. This is due to the corner present in the altitude and angle of attack command histories. By smoothing these corners in the commands, the control surface limit violation can be avoided.

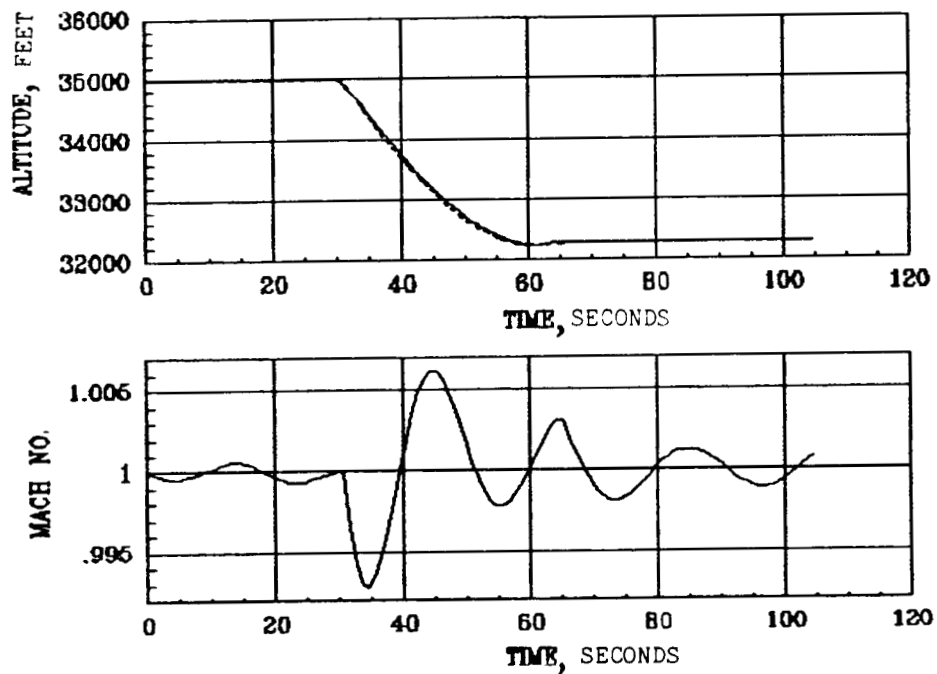


Figure 5-20. Altitude and Mach Number Evolution Along the Constant Throttle Windup Turn Flight Test Trajectory.

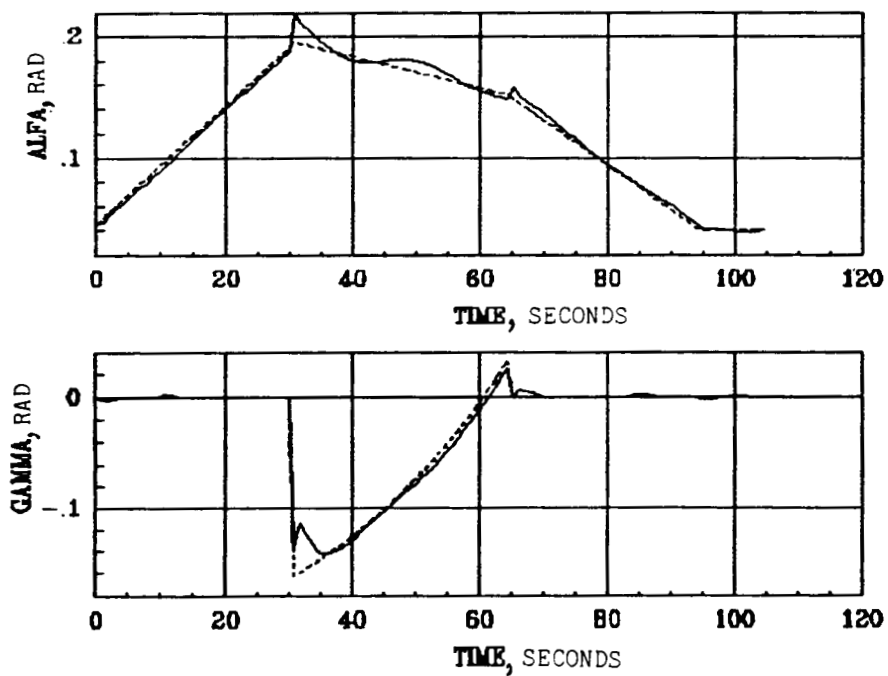


Figure 5-21. Angle of Attack and Flight Path Angle Evolution Along the Constant Throttle Windup Turn Flight Test Trajectory.

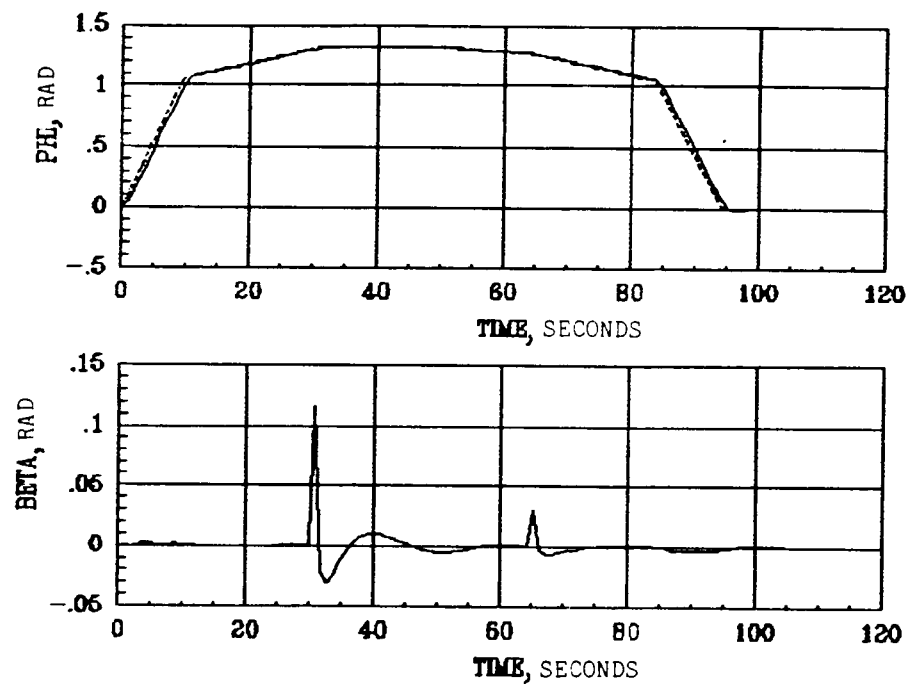


Figure 5-22. Roll Altitude and Angle of Side Slip Evolution Along the Constant Throttle Windup Turn Flight Test Trajectory.

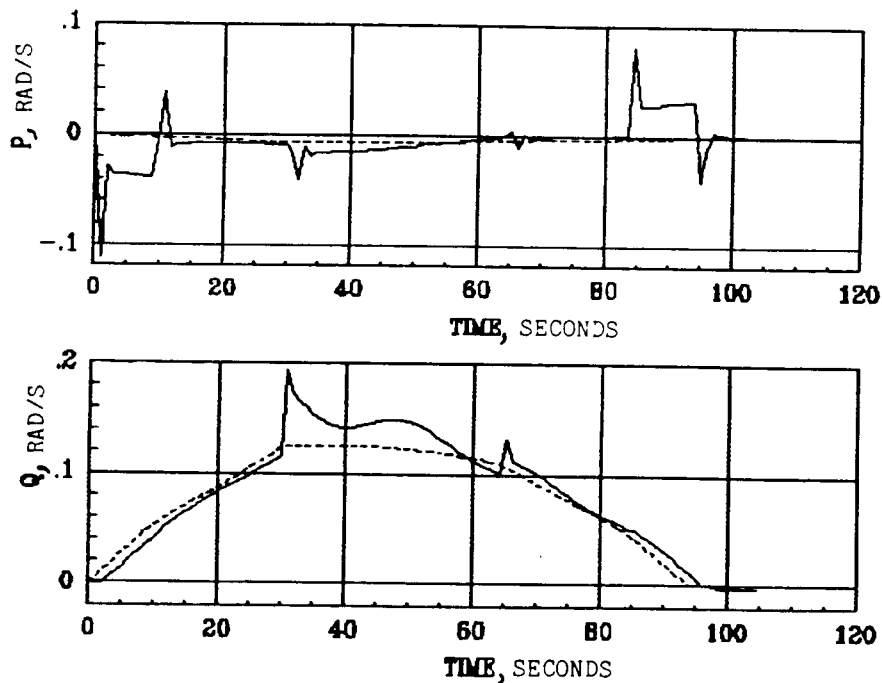


Figure 5-23. Roll and Pitch Body Rate Evolution Along the Constant Throttle Windup Turn Flight Test Trajectory.

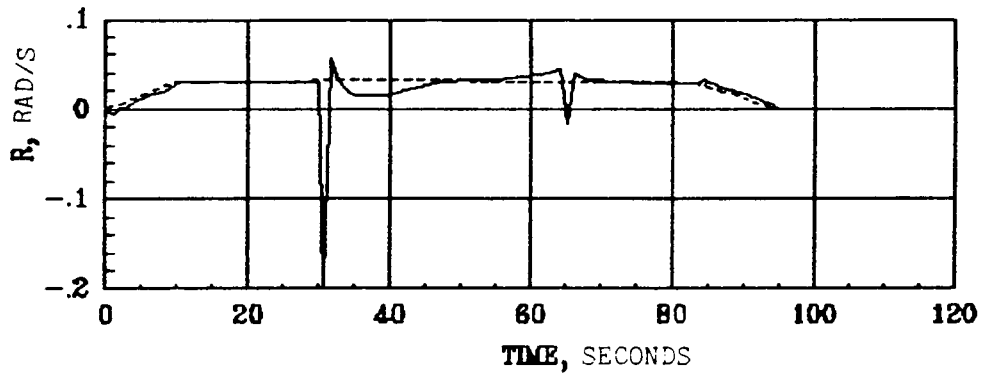


Figure 5-24. Yaw Body Rate Evolution along the Constant Throttle Windup Turn Flight Test Trajectory.

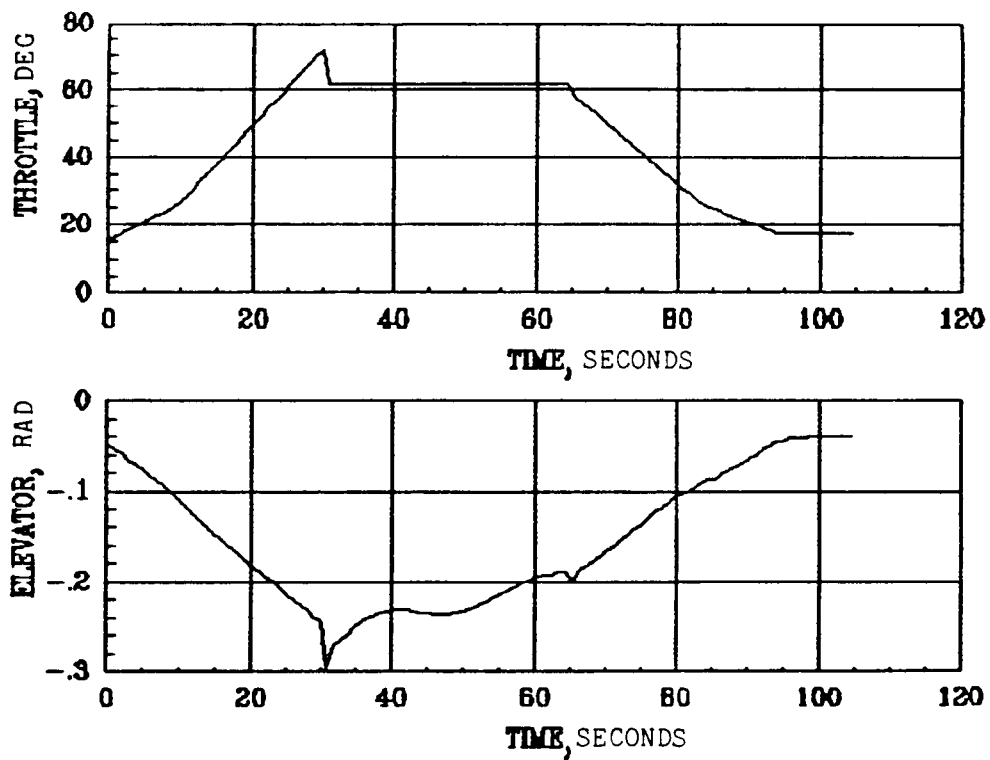


Figure 5-25. Throttle and Elevator Deflection along the Constant Throttle Windup Turn Flight Test Trajectory.

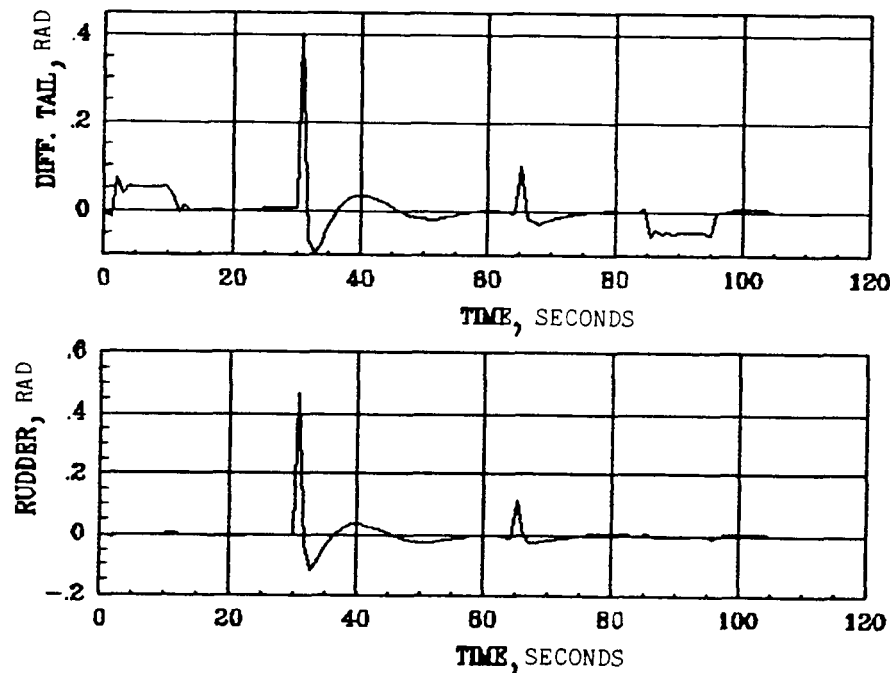


Figure 5-26. Differential Tail and Rudder Deflection along the Constant Throttle Windup Turn Flight Test Trajectory.

#### 5.3.7 Constant Dynamic Pressure - Constant Load Factor Maneuver

As noted in the maneuver modeling section, this maneuver can be ascending or descending based on the required Mach rate. The controller performance along an ascending constant dynamic pressure-constant load factor trajectory is given in Figs. 5-27 through 5-34. This trajectory consists of three phases. The first phase begins with initial conditions chosen to obtain the desired dynamic pressure. Next the aircraft is placed in a turn to generate the required load factor. In the present case, a load factor of 4 was employed. Next, the desired Mach rate is initiated and the constant dynamic pressure-constant load factor trajectory is executed. In the present case, in order to achieve a Mach rate of 0.0067/second, the aircraft had to climb from 35000' altitude to 43000' altitude in 30 seconds. These histories are given in Fig. 5-27. The control surface deflections are well within the saturation limits except at the points where the altitude and Mach number commands contains corners. The dynamic pressure history given in Fig. 5-34 was maintained within 4% of the required value.



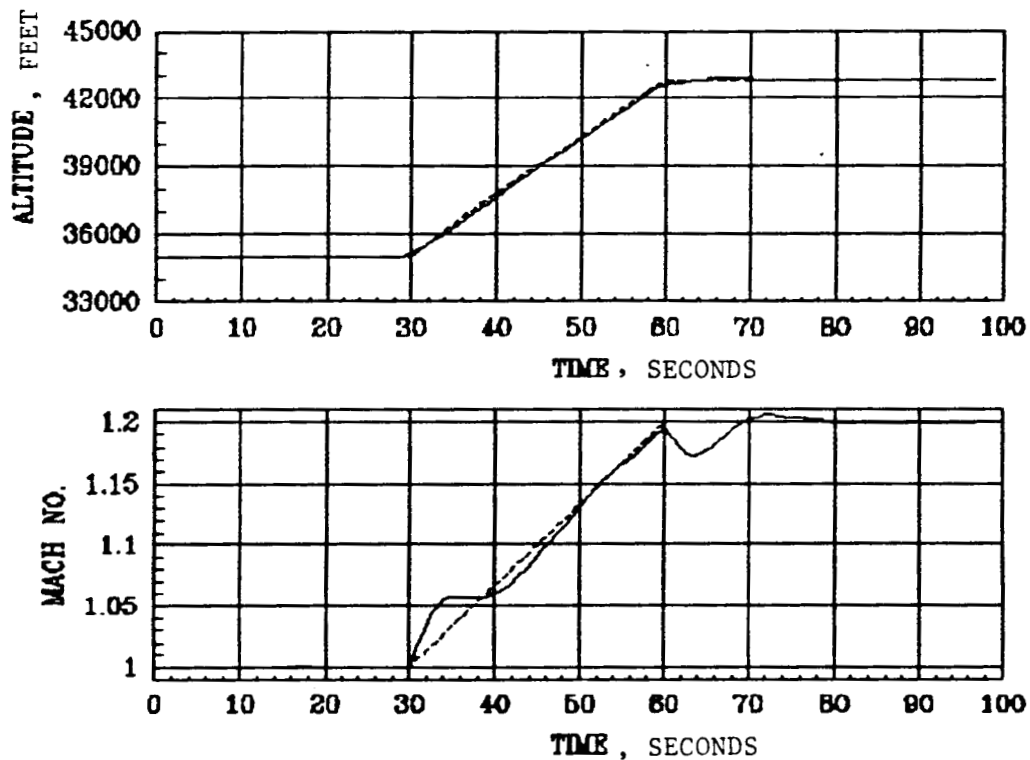


Figure 5-27. Altitude and Mach Number Evolution Along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

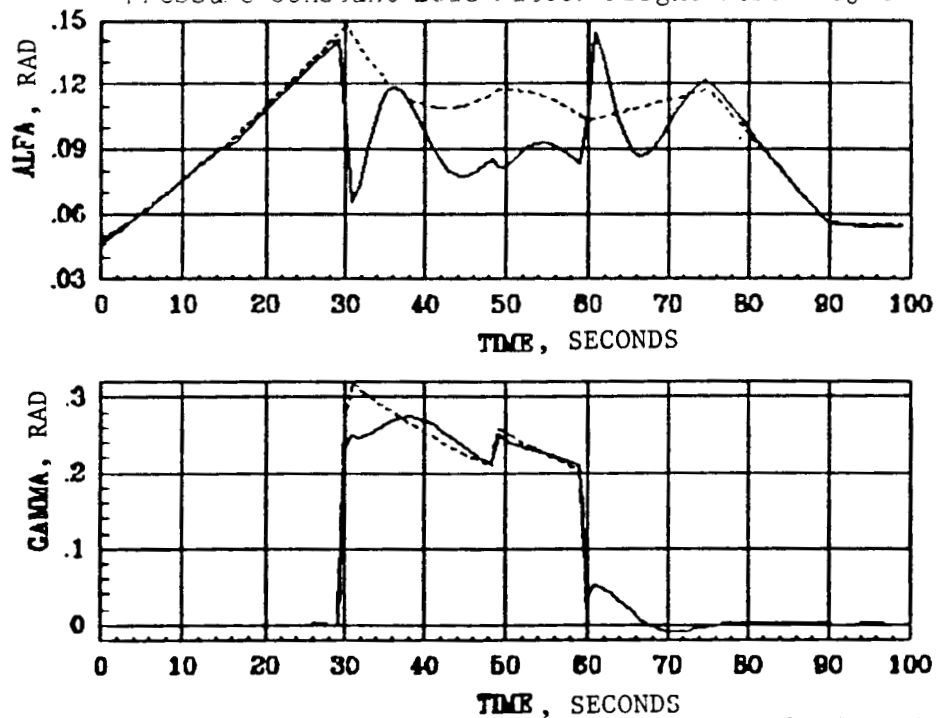


Figure 5-28. Angle of Attack and Flight Path Angle Evolution along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

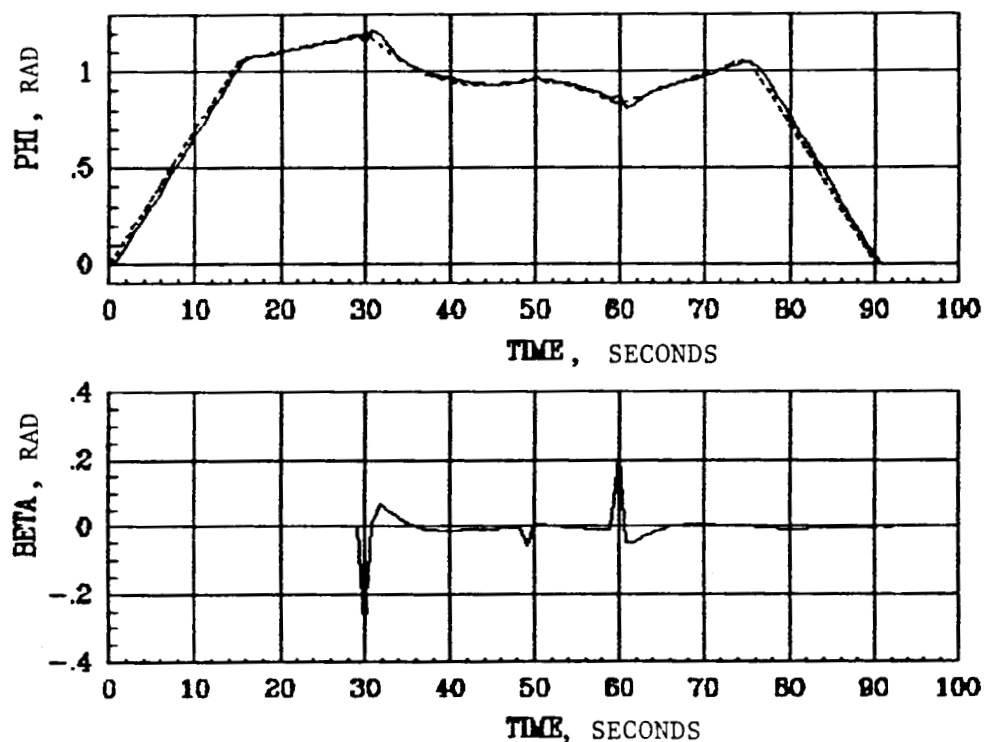


Figure 5-29. Roll Attitude and Angle of Side Slip Evolution along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

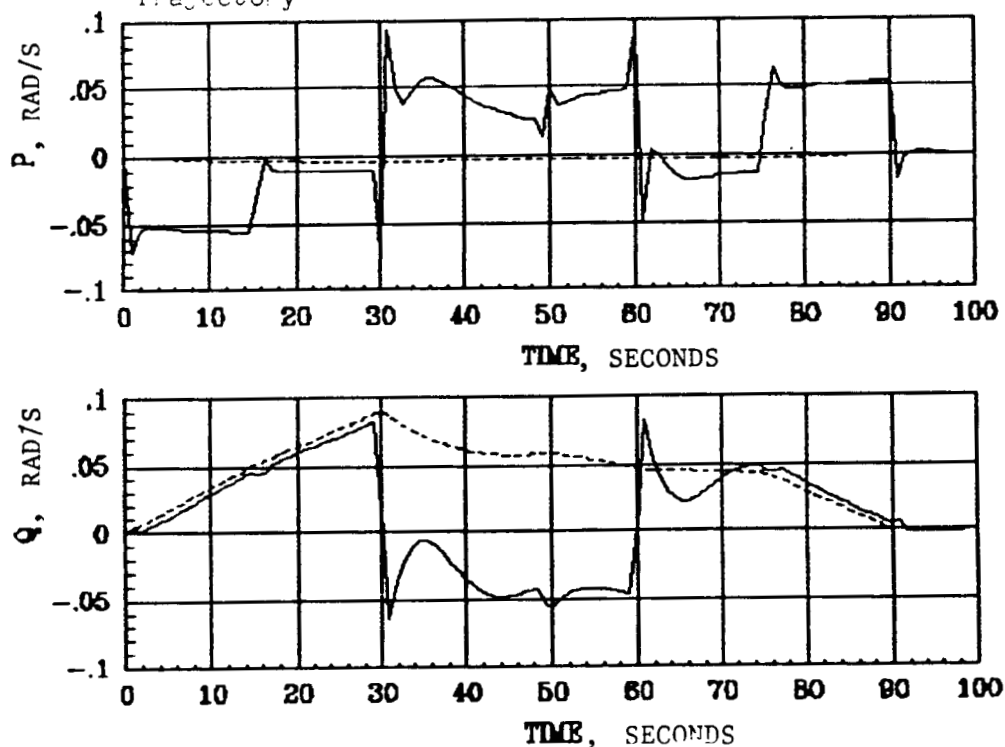


Figure 5-30. Roll and Pitch Body Rates along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

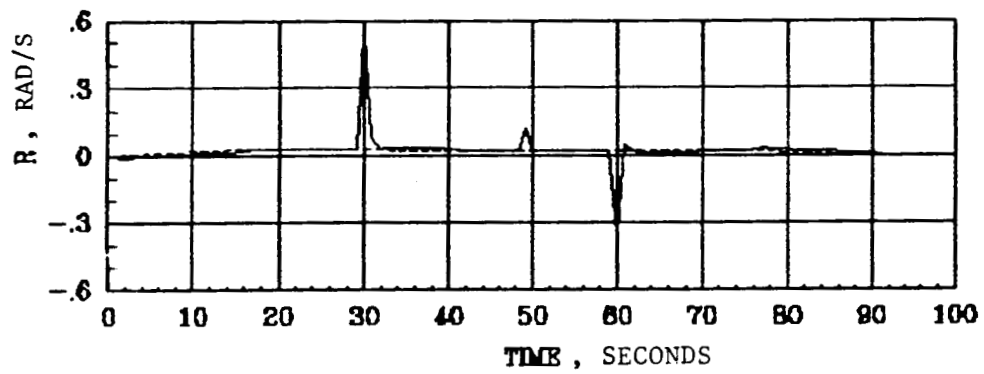


Figure 5-31. Yaw Body Rate along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory.

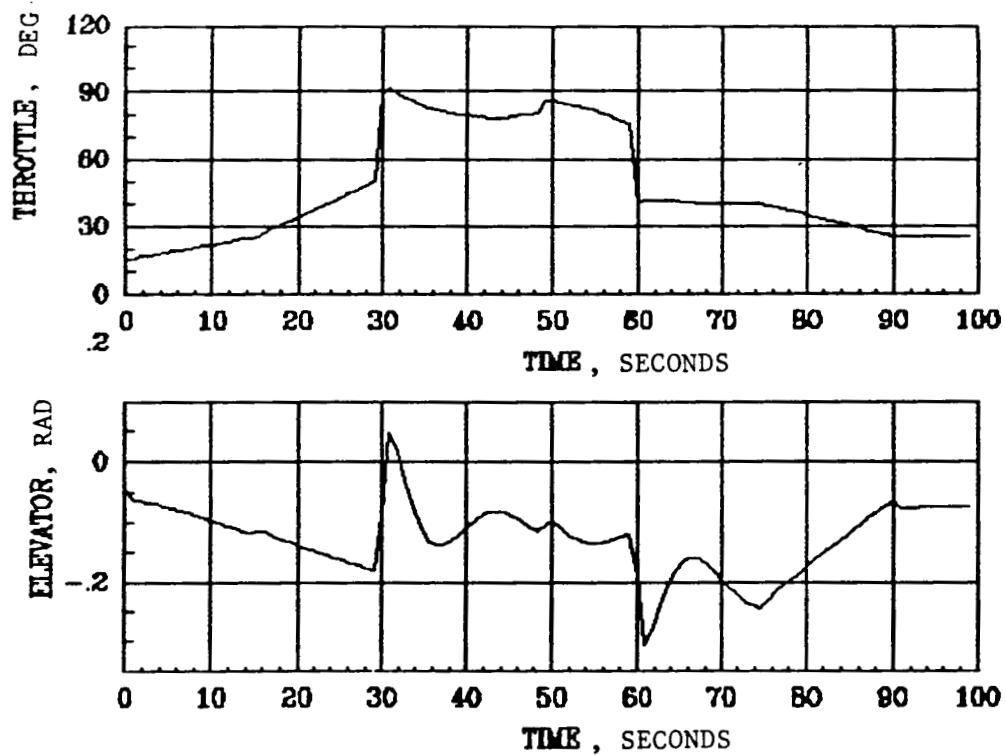


Figure 5-32. Throttle and Elevator Deflection Along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory.

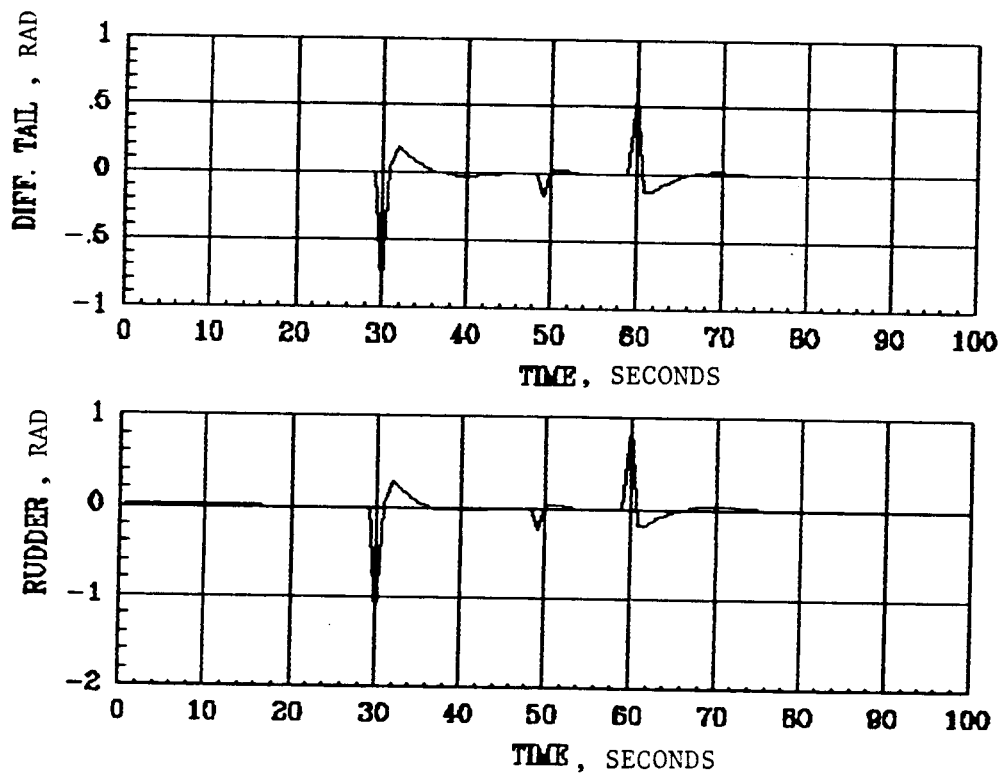


Figure 5-33. Differential Tail and Rudder Deflection along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

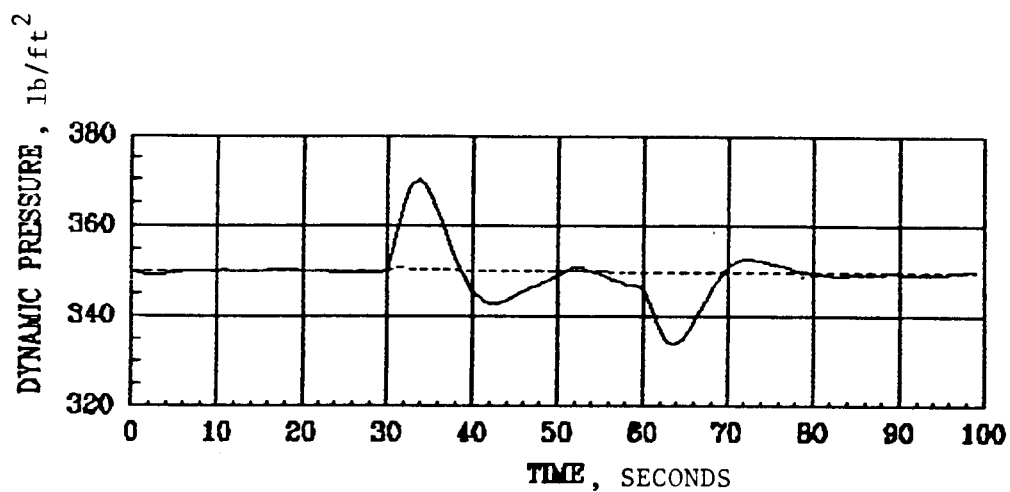


Figure 5-34. Dynamic Pressure along the Constant Dynamic Pressure Constant Load Factor Flight Test Trajectory

### 5.3.8 Constant Reynold's Number - Constant Load Factor Maneuver

This maneuver sequence is identical to the constant dynamic pressure-constant load factor trajectory. The controller performance for a negative Mach rate - Constant Reynolds' number-constant load factor trajectory is given in Figs. 5-35 through 5-42. Note that the Reynold's number given in Fig. 5-42 should be multiplied by the characteristic diameter to obtain the actual Reynold's number. From the performance results given in Fig. 5-40 and 5-41, it can be seen that the control surface deflections momentarily violate the saturation limits at the points corresponding to the corners in the commanded values. Throughout the trajectory, Reynold's number is within 1.5% of the required value.

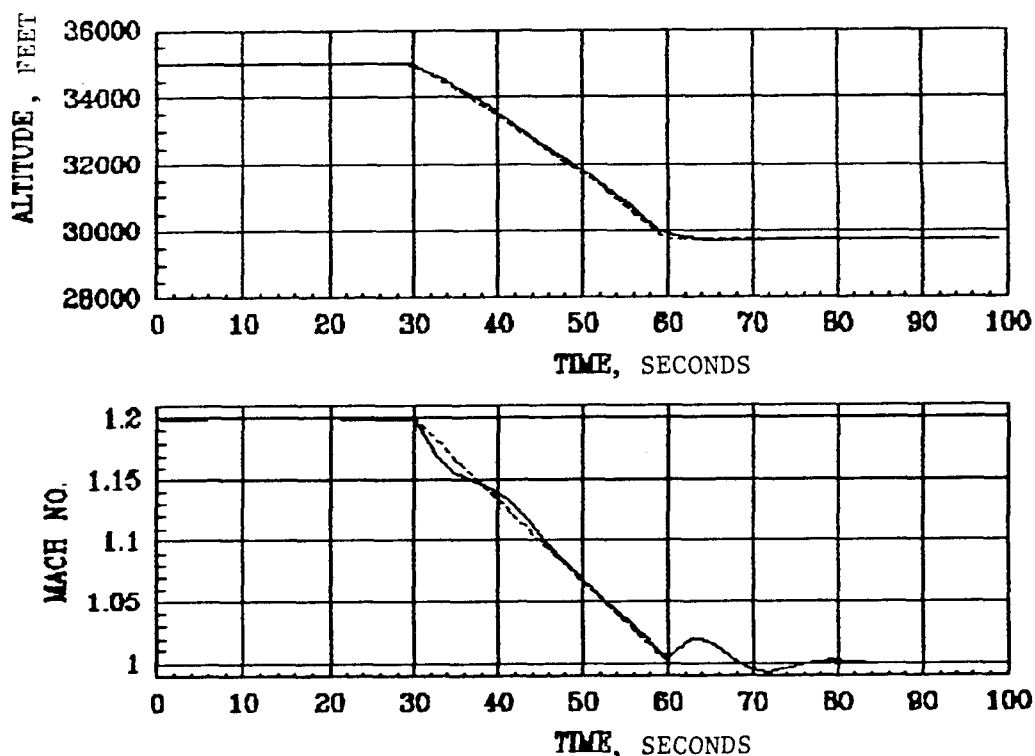


Figure 5-35. Altitude and Mach Number Evolution Along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory.

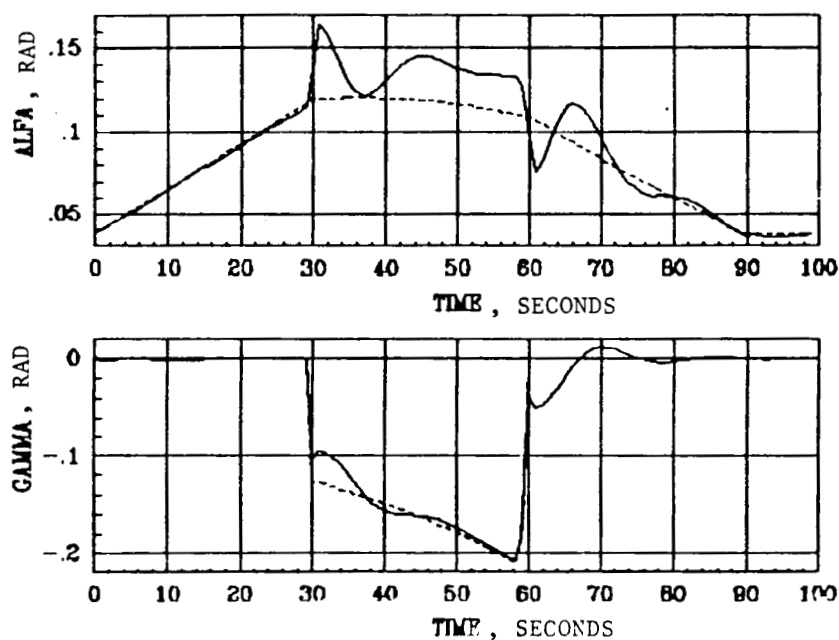


Figure 5-36. Angle of Attack and Flight Path Angle Evolution along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory

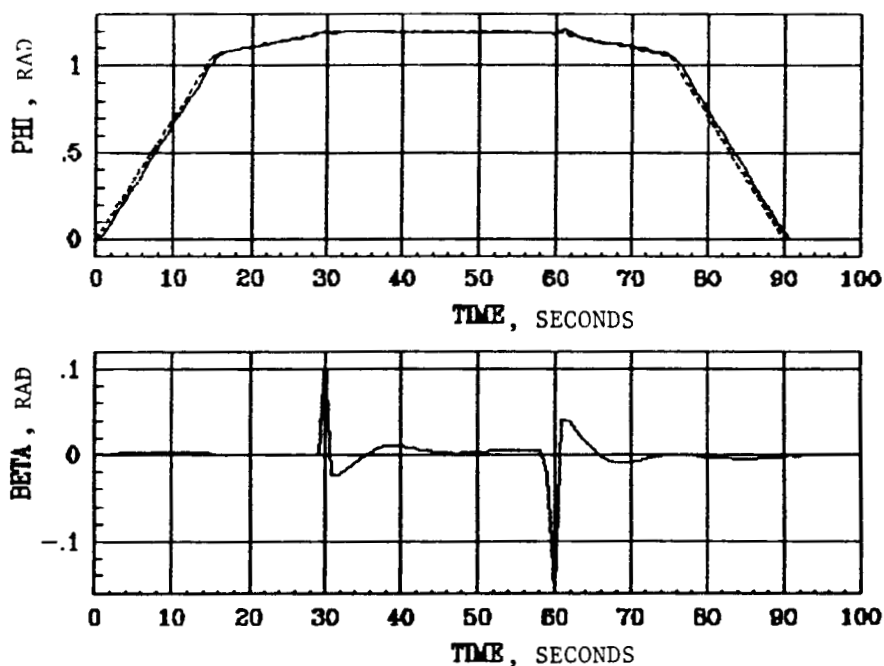


Figure 5-37. Roll Attitude and Angle of Side Slip Evolution Along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory.

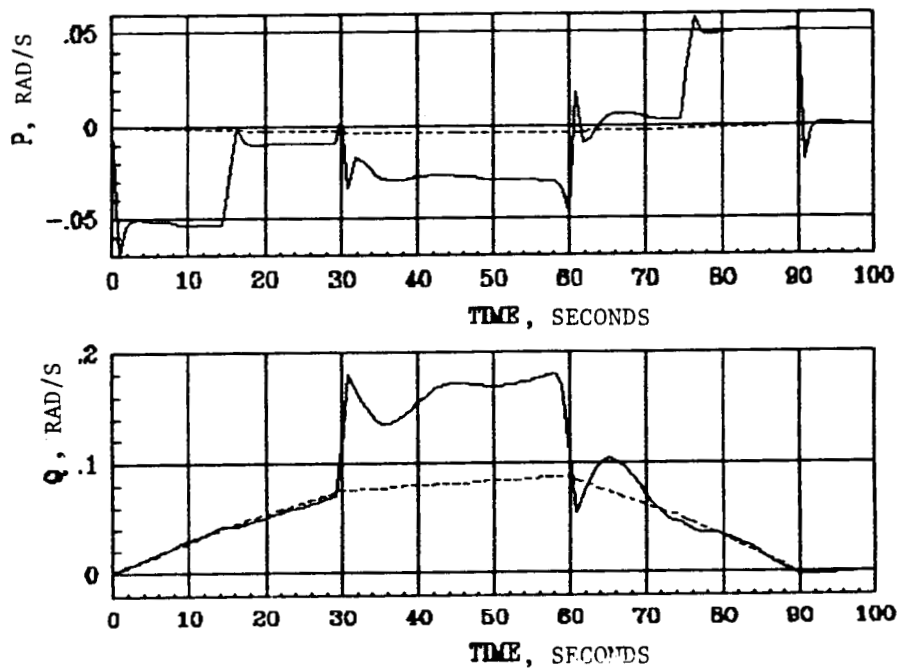


Figure 5-38. Roll and Pitch Body Rates along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory.

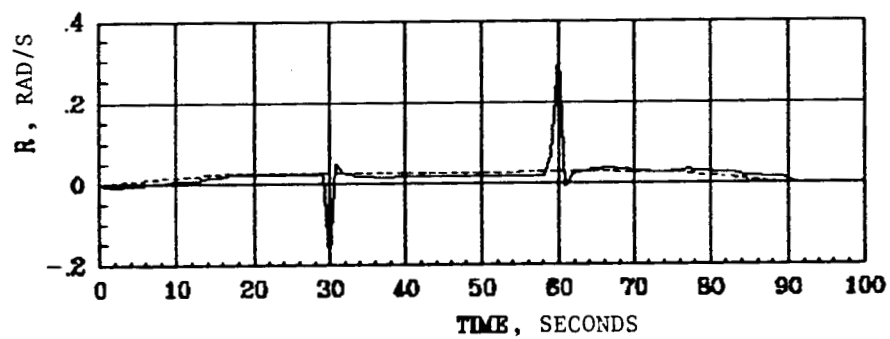


Figure 5-39. Yaw Body Rate along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory.

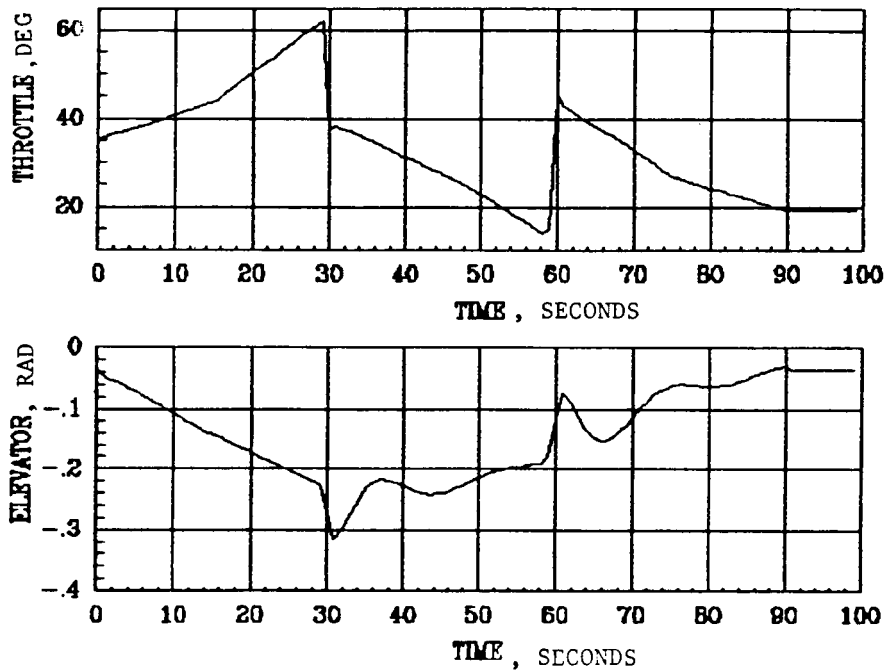


Figure 5-40. Throttle and Elevator Deflection along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory

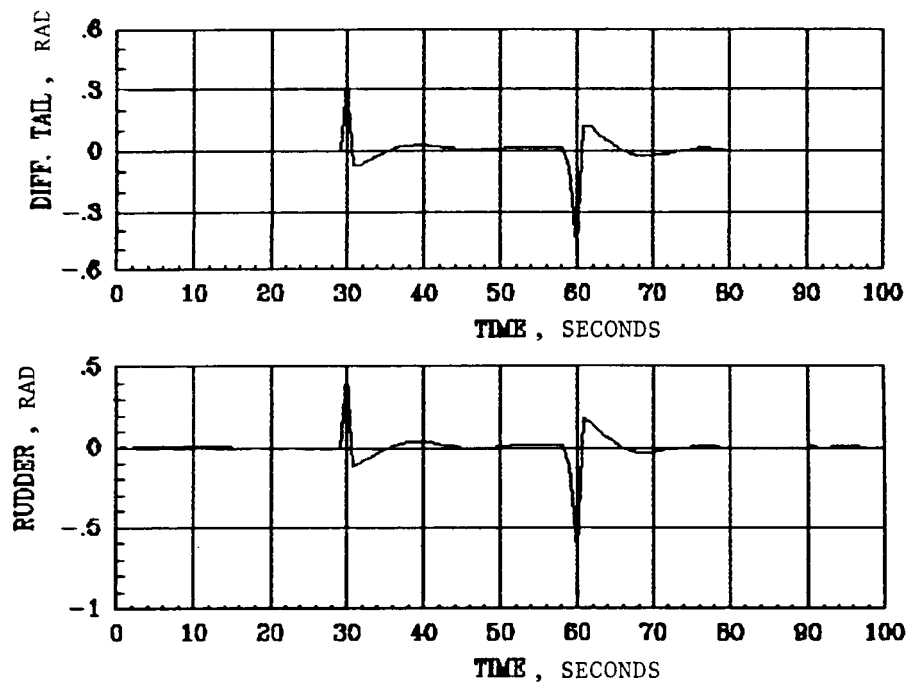


Figure 5-41. Differential Tail and Rudder Deflection Along the Constant Reynold's Number Load Factor Flight Test Trajectory.



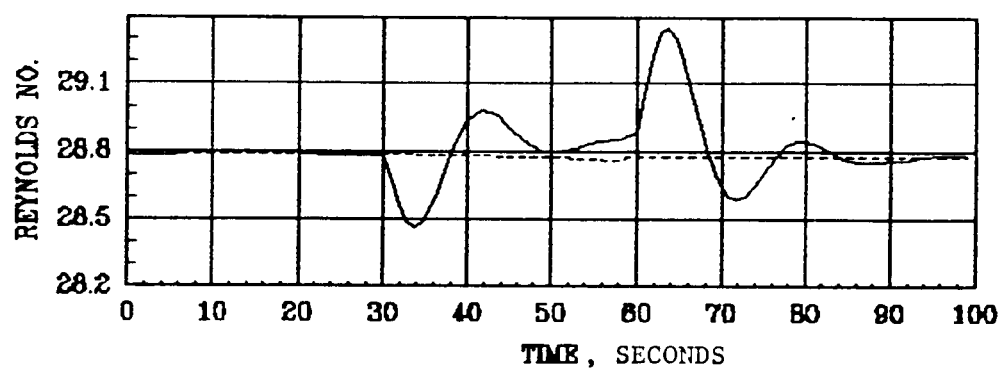


Figure 5-42. Reynold's Number Along the Constant Reynold's Number Constant Load Factor Flight Test Trajectory.

## SECTION 6

### SUMMARY AND FUTURE WORK

This report summarizes the analysis done in the first phase of a study for developing and validating a flight test trajectory controller for eight maneuvers. System modeling, control design results, control design technique evaluation, validation and software deliverable results and conclusions are summarized here. This final section describes future work which will build upon this study in the further validation and flight testing of these flight test trajectory controllers.

#### 6.1 SYSTEM MODELING

With newly developed linearization tools by NASA Ames Dryden Flight Research Facility, the linear airframe models are straight forwarded to obtain. Command augmentation system CAS models are important and although they complicate the maneuver autopilot designs, they are essential since the maneuver autopilot must work through the CAS in the manned simulation or in the F-15 itself.

The output analytical constrained relations developed in section 3 which discusses maneuver modeling were found to be fundamental in achieving a consistent maneuver autopilot. The nonlinear aerodynamics can be effectively reduced to a reference command table by appropriate linearizations throughout the flight envelope. A small number of linear perturbation models can be used to decompose the eight desired maneuvers into a few sets of linear perturbation equations.

#### 6.2 CONTROL DESIGN RESULTS

Output feedback controllers with appropriate integral aerostates were found to be the simplest form of feedback controllers. Integral aerostates must be chosen carefully to avoid controllability problems which are

problematic in the design stage. The use of a guaranteed stability margin in the control design technique was found to be a powerful method when working with the high order augmented and coupled models. It was found that a five second response time with a minimal amount of overshoot was straightforward to achieve at all design points.

### 6.3 CONTROL DESIGN TECHNIQUE EVALUATIONS

The eigenstructure assignment technique was found to work well on a bare airframe but with a complex augmented model was not useful as a design technique because there is no procedure to perform a converging iterative design approach.

The minimum error excitation output feedback method with a preliminary guaranteed stability margin full state design worked well at all flight conditions.

### 6.4 MANEUVER AUTOPILOT VALIDATION

The maneuver autopilots were validated in a linear simulation. The tracking response for reasonably high performance maneuvers were found to be quite acceptable, and except for exceeding the control authority at isolated points, is ready for further testing in a full nonlinear simulation.

### 6.5 SOFTWARE DELIVERABLES

The tools used in model development control design evaluation and design throughout the envelope as well as validation were made available to NASA Ames Dryden Flight Research Facility. These included a time varying simulation mechanized in ISI's MATRIX<sub>x</sub> SYSTEM\_BUILD (see Appendix A). In addition a stand alone program which performs a three dimensional

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interpolation in state variables and converts this through the interpolation into a 1D table as a function of time for a specific maneuvers was developed and also delivered to NASA. The 1-D interpolation in time can then be used in SYSTEM\_BUILD to mechanize a time varying simulation. Documented command files were delivered for the model generation, control law design, and construction of linear simulation using a single model throughout the flight maneuver. Command files for building, loading the data and executing a time varying simulation model were also provided.

## 6.6 FUTURE WORK

The next phase of this study will involve validation of flight test trajectory control laws in a batch nonlinear simulation. The gain scheduled linear perturbation controllers developed in this study Are currently ready for validation in such a simulation. The fineness of the discretization both of the reference commands and linear perturbation models may need to be revised in the next phase. An open-loop simulation of the reference control values will give an initial check of the accuracy of our nonlinear tabular model. The nonlinear simulation provided by NASA Ames Dryden Flight Research Facility will include both the CAS model as well as the airframe dynamics and Sensor models. The structure of the nonlinear control law, while only outlined here, was developed to a point where with some symbolic or algebraic manipulation by hand, it can be mechanized on the nonlinear simulation. The advantage of this control law is that a single set of gains will work for all maneuvers throughout the envelope. Such a transformed linear system can be easily controlled and the resulting gains back transformed with the nonlinear equations to give a nonlinear control law.

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APPENDIX A

FLIGHT TEST TRAJECTORY CONTROLLER SYNTHESIS

WITH CONSTRAINED EIGENSTRUCTURE ASSIGNMENT



## INTRODUCTION:

This Appendix gives the details of research conducted on the eigenstructure assignment. The material in the Appendix A-I was presented as a paper in the 1985 American Control Conference at Boston. Appendix A-II and A-III give the aircraft - CAS models and the constrained eigenstructure designs at two flight conditions. Appendix A-IV gives the desired eigenvalues and eigenvectors used in the synthesis.

The aircraft - CAS model used here is of the form

$$\dot{x} = Fx + Gu$$

$$y = Hx$$

where:

the state variables  $x$  are the perturbed values of

- $V$ , total airspeed
- $\alpha$ , Angle of attack
- $q$ , pitch body rate
- $\theta$ , pitch attitude
- $\beta$ , angle of sideslip
- $p$ , roll body rate
- $r$ , yaw body rate
- $\phi$ , roll attitude
- $h$ , altitude
- engine actuator state and 21 CAS states

$U$ : the control vector consists of perturbed values of throttle, elevator, differential tail and rudder.

y: the measurement vector and consists of the perturbed values of

$\dot{p}$ : roll angular acceleration

$A_n$ : normal acceleration

q: pitch rate

$\dot{q}$ : pitch angular acceleration

p: roll rate

$A_{y,i}$ : y-body acceleration, not at the vehicle center of gravity

$\dot{r}$ : yaw angular acceleration

r: yaw rate

h: altitude

M: mach number

$\alpha$ : angle of attack

n: load factor

$\phi$ : roll attitude

$\theta$ : pitch attitude

$\dot{h}$ : altitude rate

UB: x-body axis velocity

$A_{nx}$ : x-body axis acceleration at vehicle C.G.

#### CONCLUSIONS:

The constrained eigenstructure assignment design procedure in its present form demands several iterations to converge to a satisfactory design and does not appear to easily yield suitable insight for output feedback design of high order multivariable systems. If a rational method to generate an achievable set of desired eigenvectors is devised, this technique will be made more attractive.

APPENDIX A-I

PAPER PRESENTED AT 1985 ACC, JUNE 19-21

BOSTON, M.A.

# FLIGHT TEST TRAJECTORY CONTROLLER SYNTHESIS

WITH

CONSTRAINED EIGENSTRUCTURE ASSIGNMENT<sup>+</sup>

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## ABSTRACT

Flight test trajectory controller synthesis using constrained eigenvalue/eigenvector assignment procedure is presented. Associated modeling problems and the difficulties encountered in employing this synthesis technique are highlighted.

<sup>+</sup>Research supported by NASA-Ames-Dryden Flight Research Facility under Contract NAS2-11877.

## 1. INTRODUCTION:

Flight test trajectory control is a technique designed to aid in the collection of large quantities of high quality data. This technique has provided the means for flying maneuvers consistently, precisely, and repeatedly from flight to flight. Two versions of these controllers have been used: a closed-loop automatic system and an open-loop system providing manual piloting information. A closed-loop system used to collect performance, pressures, and loads data from the highly maneuverable aircraft technology (HiMAT) vehicle is described in [1]. The application of the open-loop system on the NASA F-111 transonic aircraft technology (TACT), F-15 airframe/propulsion system interaction studies, and F-15 shuttle tiles test programs are given in [2].

Originally, the open-loop flight-test-trajectory guidance algorithms were developed on-line, in a piloted simulation using cut-and-try techniques that was not only man power intensive, but often produced less than desirable controllers. A closed-loop system designed using one-loop-at-a-time classical design approach is documented in [3]. Full-state feedback approach for closed-loop system design using Linear quadratic synthesis is described in [4]. Both these approaches have limitations in terms of design methodology and controller complexity.

The research currently underway includes an exploration of various multivariable synthesis techniques for this problem. The first approach considered is that of constrained eigen value/eigen vector assignment [5]. A primary goal is to develop controllers based on output feedback so as to decrease controller complexity and to enhance robustness. The objective of this paper is to point out strengths and weakness of this technique when used in a, realistic, relatively a large problem such as required for flight test trajectory controllers.

## 2. THE FLIGHT TEST TRAJECTORY CONTROL PROBLEM:

Flight test trajectories are flown to evaluate an aircraft within its known operational envelope and to explore the boundaries of its capabilities. This makes the flight test trajectory control a very demanding task. Control systems designed for this purpose must not only operate satisfactorily in terms of keeping the flight test variables within acceptable tolerance but should also be reasonably insensitive to model parameter variations.

An approach to closed-loop flight test trajectory controller synthesis consists of linearizing the aircraft model at several flight conditions about the flight test trajectory and designing multivariable controllers, which in some sense minimizes the deviations from the reference path. Time varying nature of the linearized aircraft model along the reference path brings about the need for scheduling gains as a function of time or as functions of some important flight variables such as dynamic pressure, Mach number etc. The gain scheduling aspects will not be pursued any further in this paper, and in all that follows, discussions will center around a linear-time-invariant aircraft model. Further, though the flight test trajectory controller is discrete, in this preliminary stage it will be assumed that the sampling rate is sufficiently high, permitting the application of continuous control design techniques.

The aircraft under consideration is a high performance fighter with command augmentation system (CAS) engaged in all the three axes. The CAS is a highly nonlinear system with saturations, multiplicative nonlinearities and gain schedules. At a particular flight condition, this system can be approximated by a linear system with "equivalent" gains derived from nonlinear simulations. The aircraft model at the same flight condition is obtained from a generic aircraft linearization code developed at NASA-Dryden Flight Research Facility [6]. The state variables used are total speed, angle of attack and angle of sideslip, pitch rate, yaw rate, roll rate, pitch attitude, roll altitude and altitude. Throttle, rudder, elevator and differential tail constitute the control variables. The engine dynamics are modeled as a first order lag. Thus, the aircraft together with CAS at a particular flight condition is a coupled 31-st order system with four controls, the aircraft model having 10 states. A block diagram of the system

with flight test trajectory controller (called the Maneuver AutoPilot here) included in the loop is shown in Figure 1. Note that MAP controls the aircraft through CAS. Due to this, even if the aircraft model is decoupled in longitudinal and lateral axes at a particular flight condition, the CAS introduces a strong coupling into the system. In concise terms, the chief objective of the present study is to synthesize the maneuver autopilot so as to effect satisfactory transient response. Six maneuvers have been analyzed, which are sketched below.

## 2.1 Level Acceleration/Deceleration

This is a wings-level, constant altitude maneuver with Mach number constant or changing at a specified rate.

## 2.2 Pushover, Pullup

This is a wings-level, constant Mach number maneuver in which angle of attack is varied a specified increment about the trim value at some specified rate.

## 2.3 Zoom and Pushover

The zoom and pushover is a wings-level, thrust stabilized less than 1g maneuver. The flight trajectory is a parabolic path with the target Mach/altitude/angle of attack point at the apex.

## 2.4 Excess Thrust Windup Turn

This is a maneuver with angle of attack linearly increasing from the wings-level trim condition to some specified final value at a specified rate. The maneuver is performed at constant altitude and constant Mach number.

## 2.5 Constant Throttle Windup Turn

This is a maneuver with angle of attack increasing linearly at a specified rate from trim to some specified final value. The maneuver is performed at a predetermined, constant thrust level. Mach number is maintained by trading potential for kinetic energy via an appropriate altitude rate.

## 2.6 Constant Reynolds Number and Constant Load Factor Trajectory

This maneuver is initiated at a predetermined load factor, Mach number, and dynamic pressure. Thus, the initiation of this maneuver is not necessarily the wings-level condition. This maneuver can be either an ascending or descending maneuver at a specified Mach number rate. Reynolds number and load factor are held constant throughout the maneuver. Altitude is gained or lost to maintain Reynolds number with changing Mach number.

The simplest of these is the level acceleration/deceleration trajectory and the MAP design for this maneuver will be used as the illustrative example in this paper.

## 3. OUTPUT FEEDBACK DESIGN:

As indicated earlier, output feedback is attractive because of simplicity. Further, gain scheduling will be essential in this situation and in order to minimize the amount of stored data, it is desirable to have a capability to impose control structural constraints.

Currently, several approaches are available for output feedback design, see Refs [5, 7-10] for example. It is not the purpose of this paper to compare and contrast these, but to evaluate a specific technique from an application point of view. Constrained eigen value/eigen vector assignment technique of [5] was used in this particular application with the hope that it would permit the generation of designs based on the classical notions of poles, zeros and their relative location in the complex plane. Additionally, the selection of this technique was motivated by the example in [5], viz, the lateral axis stability Augmentation System for the L1011



airplane. Salient features of this approach are outlined in the following for clarity of the subsequent discussions.

### 3.1 Constrained Eigen Value Vector Assignment [5]:

Consider the linear time invariant system

$$\dot{x} = Ax + BU$$

$$y = Cx$$

with  $X \in R^n$ ,  $U \in R^m$ ,  $y \in R^r$  and  $A$ ,  $B$ ,  $C$  are real constant matrices of compatible dimensions.

It is desired to design a feedback controller of the form

$$U = Fy$$

with a structural constraint that some specified elements of  $F$  satisfy

$$f_{ij} = 0$$

It is assumed that the system can be stabilized with given outputs  $y$ , and that any dynamic compensators required have been appended to the original system.

The design problem is: given a self conjugate set of scalars  $\{\lambda_i^d\}$   $i = 1, 2 \dots r$ , and a corresponding self conjugate set of  $n$  vectors  $\{v_i^d\}$   $i = 1, 2 \dots r$ , and a given controller structural constraint, find a real  $m \times r$  matrix  $F$  such that  $r$  of the eigen values of  $A + BFC$  are "close" to the set  $\{\lambda_i^d\}$  and the corresponding eigen vectors of  $A + BFC$  are "close" to the respective member of  $\{v_i^d\}$ .

Note that if there are no constraints on the feedback matrix, it is feasible to place  $r$  desired eigen values exactly.

However, one is not at liberty to place  $v_1, v_2 \dots v_r$  by an arbitrary set of desired closed loop eigen vectors  $\{v_i^d\}$   $i = 1, 2 \dots r$  since this set might not belong to the set of assignable closed loop eigenvectors. It then becomes necessary to find some approximation to the set  $\{v_i^d\}$  which is assignable, yet "close" in some sense to the original  $\{v_i^d\}$ . It will be seen subsequently that this is the main difficulty in the application of this technique to a complex problem such as flight test trajectory controller design.

Complete details of this approach can be found in [5]. It is perhaps of interest to note that in addition to the capability to handle controller structure constraints, this technique can be modified to accept partial specification of eigen vectors. This feature can be valuable in high order systems. In summary, to employ this synthesis approach, the following are required.

- (i) Based on the practical aspects of the problem, choose a minimal set of measurements which will permit the designer to achieve the desired performance. Introduce dynamic compensators such as integral feedbacks, lead-lag networks, etc., based on experience.
- (ii) Choose a set of desired eigenvalues and eigenvectors equal to the number of outputs.

While the selection of desired eigenvalues is often apparent in a given problem, the desired eigenvectors are difficult to select. A piece of information which might serve as a guide in this selection is the fact the closed loop eigenvectors  $v_i$  corresponding to the closed loop eigenvalues  $\lambda_i$  must lie in the subspace spanned by the columns of the matrix  $(\lambda_i I - A)\bar{B}^{-1}$ . Further, the matrix  $C V$ ,  $V = \{v_1, v_2 \dots v_r\}$  should be invertible. If the complete specification of desired eigenvectors are not feasible, a rough rule is to pick the entries in these eigenvectors as zeros or ones based on whether a particular measurement needs to contribute to a particular mode or not. In any case, constructing a set of desired eigenvectors that are assignable constitutes the most difficult part of this design technique.

#### 4. FLIGHT TEST TRAJECTORY CONTROLLER SYNTHESIS:

As an illustrative example, constrained eigenstructure assignment technique is next used in the design of level acceleration/deceleration flight test trajectory. Alternate design approaches have been used for this maneuver and hence comparative evaluation was feasible. The level acceleration/deceleration maneuver requires the roll attitude to be maintained zero and the altitude to be held constant. The mach number should be maintained constant or should change at a specified rate. To ensure zero steady state errors, integral feedbacks are first introduced in the altitude and mach number channels. Since the mach number command can be a ramp, an additional integrator is required in this channel to decrease tracking errors. This, however, was not done because the tracking errors with a single integral feedback has been found to be within acceptable values.

The desired eigenvalues to be used in the design are clear at the outset, based on the four modes for aircraft model, viz, the phugoid, short period, dutch roll and roll convergence. The controller structure constraint in this flight maneuver is that the errors in lateral channel will be corrected using rudder and differential tail, while the errors in longitudinal channel will be corrected using elevator and throttle. In view of the time varying nature of the model, one would like to pick a set of eigenvalues and eigenvectors at a particular flight condition through extensive design iterations and then attempt to use these at other flight conditions.

The choice of desired eigenvectors is not clear at this point. Three approaches were tried with varying degrees of success. These are sketched in the following.

##### 4.1 Minimally Restructured Eigenassignment

Since it is known that the closed loop eigenvectors lie in a subspace spanned by the columns of  $(\lambda_i I - A) \bar{B}^1$ ,  $i = 1, \dots, n$ , linear combinations of these vectors were used as desired eigen vectors. The weights to be used in generating these linear combinations were constructed from the additional information that unassigned eigenvectors should be close to their open loop values in a least square sense.

## 4.2 Decoupling Eigenassignment

A variation to the above was attempted next. Since we are interested in having the least cross axis coupling in the controller as possible, the desired eigenvectors in the longitudinal channel may be chosen so that the responses from lateral channels are blocked, i.e. select eigenvectors as

$$\left[ \begin{array}{c|c} (\lambda_i I - A) & B \\ \hline C_{LAT} & 0 \end{array} \right] \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V_d = \sum_{i=1}^n \alpha_i v_i$$

$V_d$  are the desired eigenvectors.  $\alpha_i$  are selected using the same criteria as 4.1. Though these two approaches could be made to work at each flight condition by iterating on the desired eigen values, they failed to easily produce a set of acceptable eigen vectors which could be used at other flight conditions. Next, partial specification of desired eigenvectors was attempted as follows:

## 4.3 Dominant Mode Eigenassignment

According to ref [5], complete specification of desired eigenvectors are neither necessary nor desirable. Depending on the states should or should not participate in a given mode, appropriate entries in the eigenvectors are made ones or zeros, leaving other entries free. With these eigenvectors, the desired eigenvalues are moved as far left from the imaginary axis as possible with least change in the location of unplaced eigenvalues. The desired eigenvectors so obtained appeared to work over most flight conditions. Note, however, that extensive iterations on the desired eigenvalues may often be required to produce a satisfactory design.

The open-loop eigenvalues for the aircraft-CAS linearized model at Mach 1.2 and 10000' altitude is given in table 1. The system has a pole on the right half of  $s$  plane. Partial specification of the desired eigenvectors are constructed next, based on the states that should or should not participate in a given measurement. The desired eigenvectors used in the present example are given in table 2. It can be seen from this table that no restriction has been placed on states associated with CAS. A manual iteration is now undertaken to determine the desired eigenvalues. Since the designer is interested in producing a stable system with as high a speed of response as possible, the eigenvalues to be moved are the ones closest to the imaginary axis. Hence, these are moved as far to the left of the imaginary axis as possible with least change in the location of other eigenvalues. Table 3 shows a set of desired eigenvectors obtained from this exercise. The output feedback gains obtained from the constrained eigenvalue/eigenvector design technique is given in table 4. Table 5 gives the closed loop eigenvalues. Comparing this with table 3 shows that the achieved eigen values are close to the desired ones.

In Figures 2 and 3, the time response of the system for a ramp mach number command are shown. A question that occurs naturally at this point is whether the design can be improved by further adjustment of desired eigen values and eigen vectors. Examination of the constrained eigen value/eigen vector approach yields no answer to this question.

## 5. CONCLUSIONS

Design of a maneuver autopilot for flight test trajectory control using constrained eigenvalue/eigenvector assignment was discussed. Difficulties encountered in the generation of desired eigenvalues and eigenvectors were outlined. This approach demands several iterations to converge to a satisfactory design and does not appear to easily give suitable insight for output feedback design of high order multivariable systems which will be used at other operating points. If a rational method to generate an achievable set of eigenvectors is devised, this technique will be made more attractive. One possibility might be to generate gradients of the eigen-system between flight conditions and include this information in the single point design technique.

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TABLE 1. OPEN-LOOP EIGEN VALUES FOR AIRCRAFT-CAS LINEARIZED MODEL AT MACH  
1.2 AND 1000'S ALTITUDE

-5.9456D+01 +2.0293D+01i	-1.2176D+01 +1.0192D+01i
-5.9456D+01 -2.0293D+01i	-1.2176D+01 -1.0192D+01i
-1.3674D+02 -3.2629D-15i	-2.9438D+00 +5.3113D+00i
-4.8879D+01 +3.0434D-17i	-2.9438D+00 -5.3113D+00i
-8.7758D+01 +5.8218D+01i	-2.3186D-01 +1.8504D-01i
-8.7758D+01 -5.8218D+01i	-2.3186D-01 -1.8504D-01i
-1.0180D+02 +5.1723D-15i	-3.4734D-01 -7.4748D-19i
-1.0000D+02 +4.2144D-15i	-6.5704D-02 -8.2699D-18i
-8.8198D+01 +3.5469D-16i	3.1435D-04 +4.3959D-03i
-2.3286D+01 +8.2436D-17i	3.1435D-04 -4.3959D-03i
-7.4117D+00 +3.3574D+01i	-5.4174D-01 +0.0000D+00i
-7.4117D+00 -3.3574D+01i	-5.0000D-01 +0.0000D+00i
-7.9725D+00 +8.2655D+00i	1.4997D-03 +0.0000D+00i
-7.9725D+00 -8.2655D+00i	-2.7600D-01 +0.0000D+00i
	-2.0000D-01 +0.0000D+00i
	-1.0000D+00 +0.0000D+00i
	-1.9200D+01 +0.0000D+00i

TABLE 2. DESIRED EIGEN VECTORS AND THEIR INTERPRETATION "99" STANDS FOR UNSPECIFIED COMPONENTS

Measurements								
$\phi$	p	h	h	$a_x$	M	$f_M$	$f_h$	
0.	0.	0.	0.	99.	99.	99.	0.	v
0.	0.	99.	99.	1.	99.	0.	99.	$\alpha$
0.	0.	99.	99.	0.	0.	0.	99.	q
0.	0.	0.	1.	99.	0.	99.	0.	e
0.	0.	0.	0.	0.	0.	0.	0.	$\beta$
99.	1.	0.	99.	0.	0.	0.	0.	P
0.	0.	0.	0.	0.	0.	0.	0.	r
1.	99.	99.	99.	0.	0.	0.	0.	$\phi$
0.	0.	1.	99.	99.	0.	99.	99.	h
0.	0.	0.	0.	99.	1.	0.	0.	Thrust Actuator
99.	99.	99.	99.	99.	99.	99.	99.	States
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	CAS States
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	0.	0.	99.	99.	1.	0.	$f_M$
99.	99.	99.	99.	0.	0.	0.	1.	$f_h$



TABLE 3. DESIRED EIGEN VALUES

-2.0000D-01 +2.0000D-01i  
 -2.0000D-01 -2.0000D-01i  
 -1.5000D-01 +1.0000D-01i  
 -1.5000D-01 -1.0000D-01i  
 -1.4000D-01 +1.4000D-01i  
 -2.5000D-01 +0.0000D+00i  
 -1.4000D-01 -1.4000D-01i  
 -2.7000D-01 +0.0000D+00i

TABLE 4. OUTPUT FEEDBACK CONTROLLER

$$\begin{bmatrix} \delta T \\ \delta e \\ \delta a \\ \delta r \end{bmatrix} = \begin{bmatrix} -2.6554D-02 & -6.7021D-02 & -4.2989D-02 & -1.1488D+02 & -1.0926D+02 & -1.3532D+03 & -1.3895D+02 & -2.7503D-03 \\ 1.3259D-01 & 4.1906D-01 & 5.9632D-02 & 2.9394D+02 & -6.4494D+00 & -4.9372D+01 & -7.9314D+00 & 5.8963D-03 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.6041D-01 & -2.3116D+00 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \phi \\ \delta p \\ \delta h \\ . \\ \delta h \\ \delta a_x \\ \delta M \\ I \delta M \\ I \delta h \end{bmatrix}$$

TABLE 5. CLOSED LOOP EIGEN VALUES

-1.3612D+02 +7.9936D-15i	-4.1391D+00 -1.4054D+01i	} desired eigenvalues
-1.1955D+02 +2.4338D-15i	-4.1391D+00 +1.4054D+01i	
-1.0000D+02 +6.1189D-16i	-3.2028D+00 +4.0631D+00i	
-8.8194D+01 +4.5058D-15i	-3.2028D+00 -4.0631D+00i	
-8.8004D+01 -5.7886D+01i	-1.0000D+00 +0.0000D+00i	
-8.8004D+01 +5.7886D+01i	-6.9439D-01 +6.7315D-01i	
-6.1080D+01 +2.1706D+01i	-6.9439D-01 -6.7315D-01i	
-6.1080D+01 -2.1706D+01i	-5.0000D-01 +0.0000D+00i	
-4.9065D+01 +1.0959D-16i	-4.7765D-01 +0.0000D+00i	
-2.6024D+01 +6.4031D-15i	-3.4069D-01 +1.1513D-17i	
-1.9200D+01 +0.0000D+00i	-2.7000D-01 +1.3002D-17i	
-5.4187D+00 +3.3073D+01i	-2.5000D-01 +1.3092D-17i	
-8.4187D+00 -3.3073D+01i	-2.0045D-01 -2.0037D-01i	
-5.0713D+00 -4.4306D+01i	-2.0045D-01 +2.0037D-01i	
-5.0713D+00 +4.4306D+01i	-1.5000D-01 -1.0000D-01i	
	-1.5000D-01 +1.0000D-01i	
	-1.4000D-01 -1.4000D-01i	
	-1.4000D-01 +1.4000D-01i	

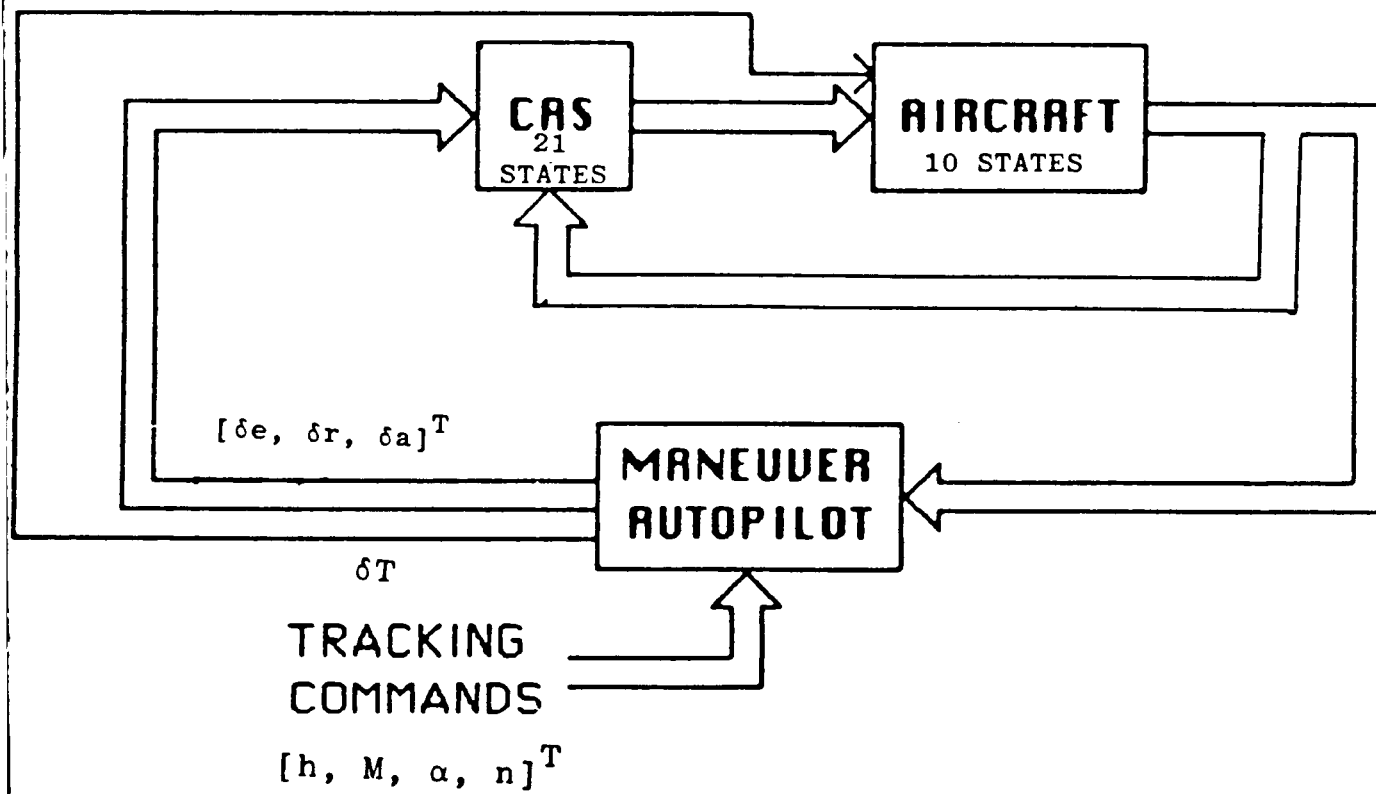


Figure 1. Flight Test Trajectory Control System

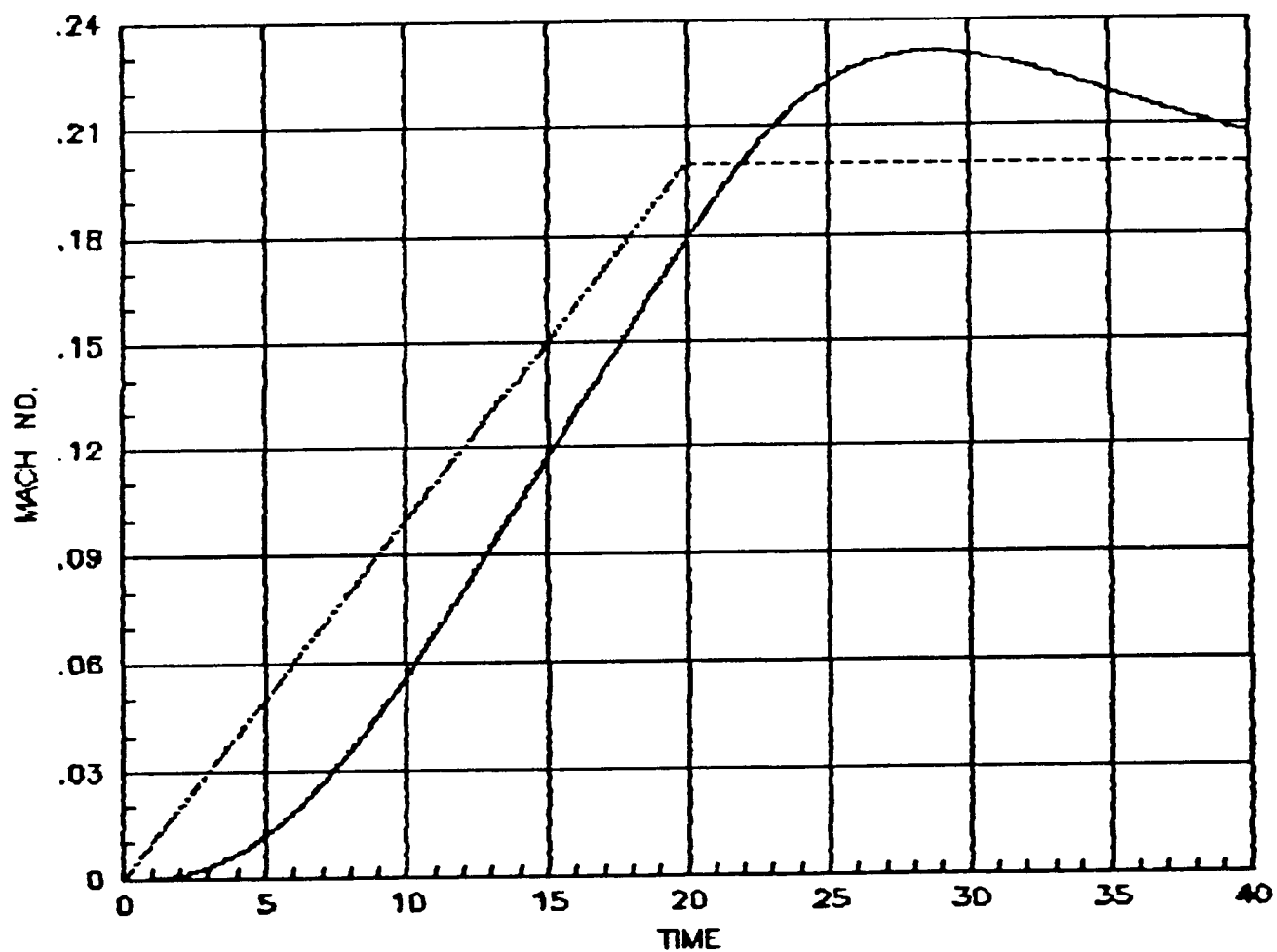


Figure 2. Mach No. vs Time Response for a Ramp Mach No. Command

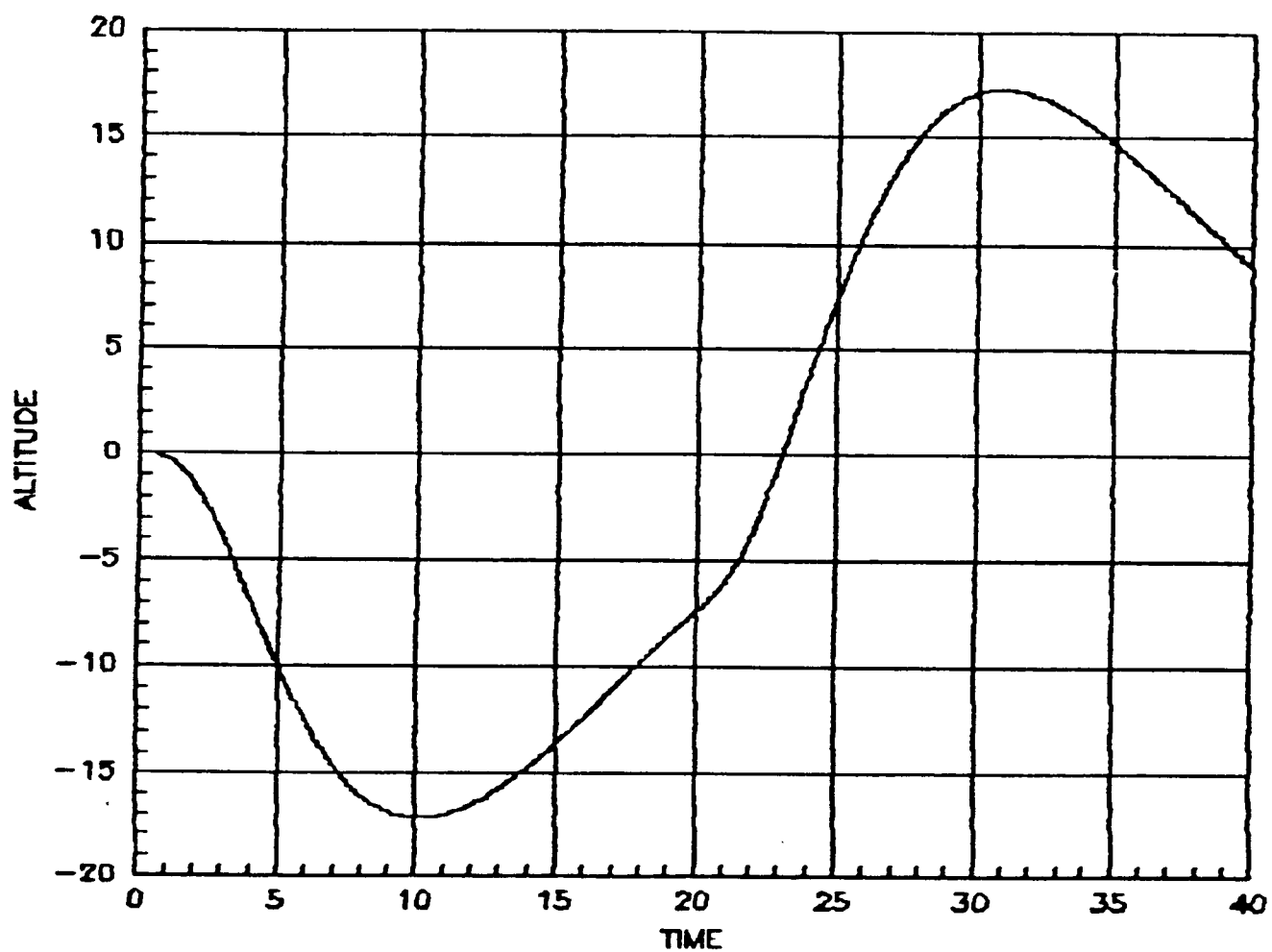


Figure 3. Altitude vs. Time Response for a Ramp Mach No. Command

APPENDIX A-II

AIRCRAFT - CAS LINEARIZED MODEL AND  
FTTC DESIGN USING EIGENSTRUCTURE ASSIGNMENT  
AT 1.2 MACH, 10000 ALTITUDE

1. F Matrix (31 x 31)

[illegible][illegible]

## F Matrix (Cont'd)

COLUMNS 13 THRU 18

[illegible]

COLUMNS 19 THRU 24

[illegible]

## F Matrix (Cont'd)

[illegible]





### 3. H-Matrix (17 x 31)

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-1.1327D+02	-5.0653D+00
1.2151D-03	9.3810D+01	0.0000D+00	-3.1062D-05	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
7.3543D-03	-1.0458D+02	-5.1574D+00	1.0534D-03	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-2.0410D+01	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	3.0049D+01	-6.0755D-03
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
9.2815D-04	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.1892D-03	9.2813D+01	0.0000D+00	-1.2611D-06	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	-1.2929D+03	0.0000D+00	1.2929D+03	0.0000D+00	0.0000D+00
9.9994D-01	-1.5381D+01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.2601D-03	-6.1083D-01	0.0000D+00	8.8325D-06	0.0000D+00	0.0000D+00

COLUMNS 7 THRU 12					
9.9565D-01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	-3.7795D-06	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	-5.0353D-08	2.9137D-06	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.0568D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	9.7656D-01	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	-3.7834D-06	0.0000D+00	0.0000D+00
0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	4.7963D-05	1.1805D-02	0.0000D+00	0.0000D+00

## H-Matrix (Cont'd)

COLUMNS 13 THRU 18

[illegible]

COLUMNS 19 THRU 24

[illegible]

## H-Matrix (Cont'd)

[illegible]

COLUMN 31

[illegible]

## Controller Design

$$U = \text{GAIN} * z$$

where

z are perturbed values of  
 $[\phi \ p \ h \ \dot{h} \ a_x \ M \ \int M \ \int h]^T$

and

$$u = [\delta T \ \delta e_{ap} \ \delta a_{ap} \ \delta \gamma_{ap}]^T$$

**GAIN =**

COLUMNS 1 THRU 6	
-2.6554D-02	-6.7021D-02
1.3259D-01	4.1906D-01
-6.0067D+00	-5.1092D+01
2.6041D-01	-2.3116D+00

COLUMNS 7 THRU 8	
-1.3895D+02	-2.7503D-03
-7.9314D+00	5.8963D-03
-1.8390D-11	3.6752D-13
-1.0162D-12	4.1311D-14

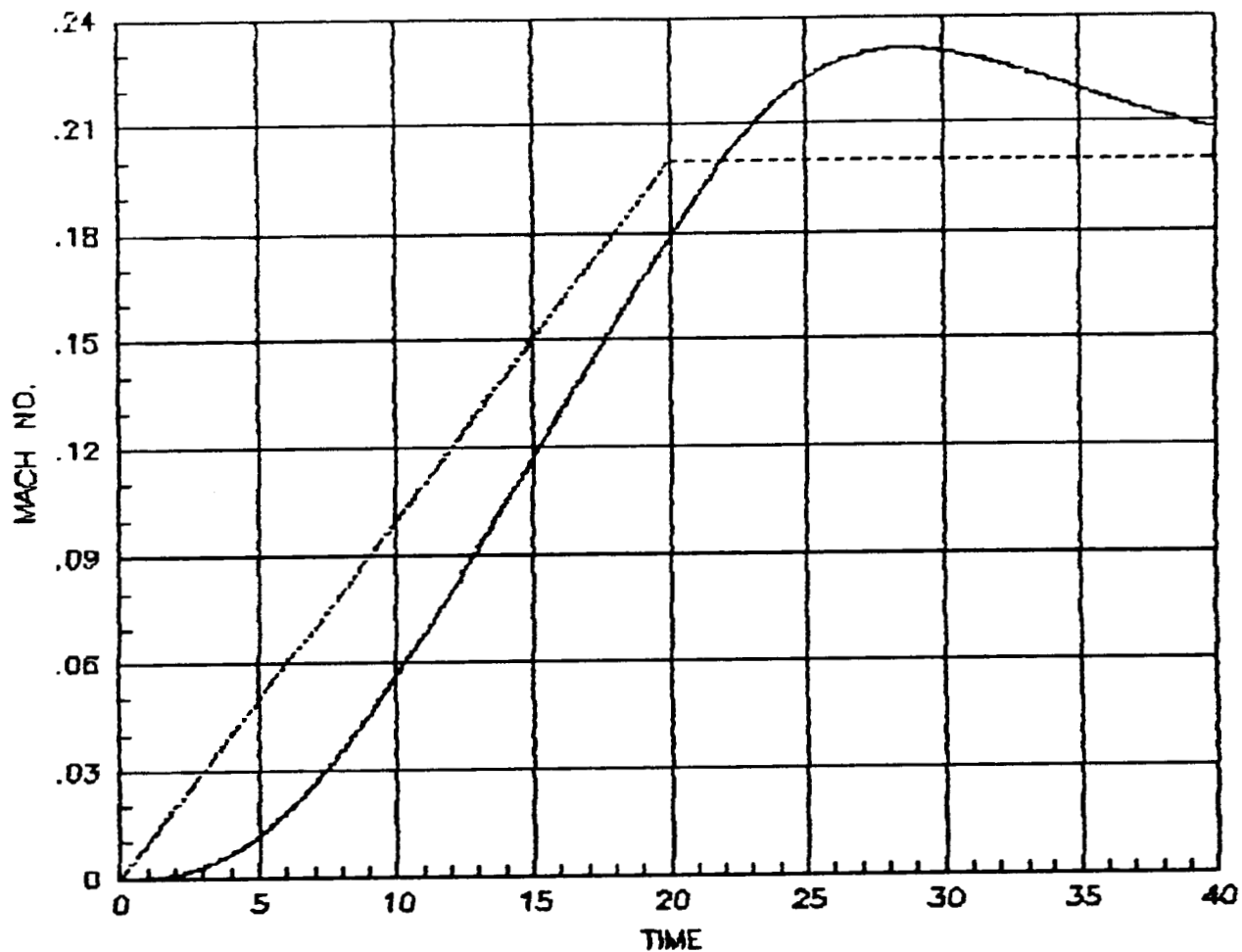
-4.2989D-02	-1.1488D+02	-1.0926D+02	-1.3532D+03
5.9632D-02	2.9394D+02	-6.4494D+00	-4.9372D+01
3.7164D-12	1.7632D-08	-7.0812D-12	6.1661D-10
4.0677D-13	1.9321D-09	-4.5292D-13	7.2911D-11

Open-loop eigenvalues:

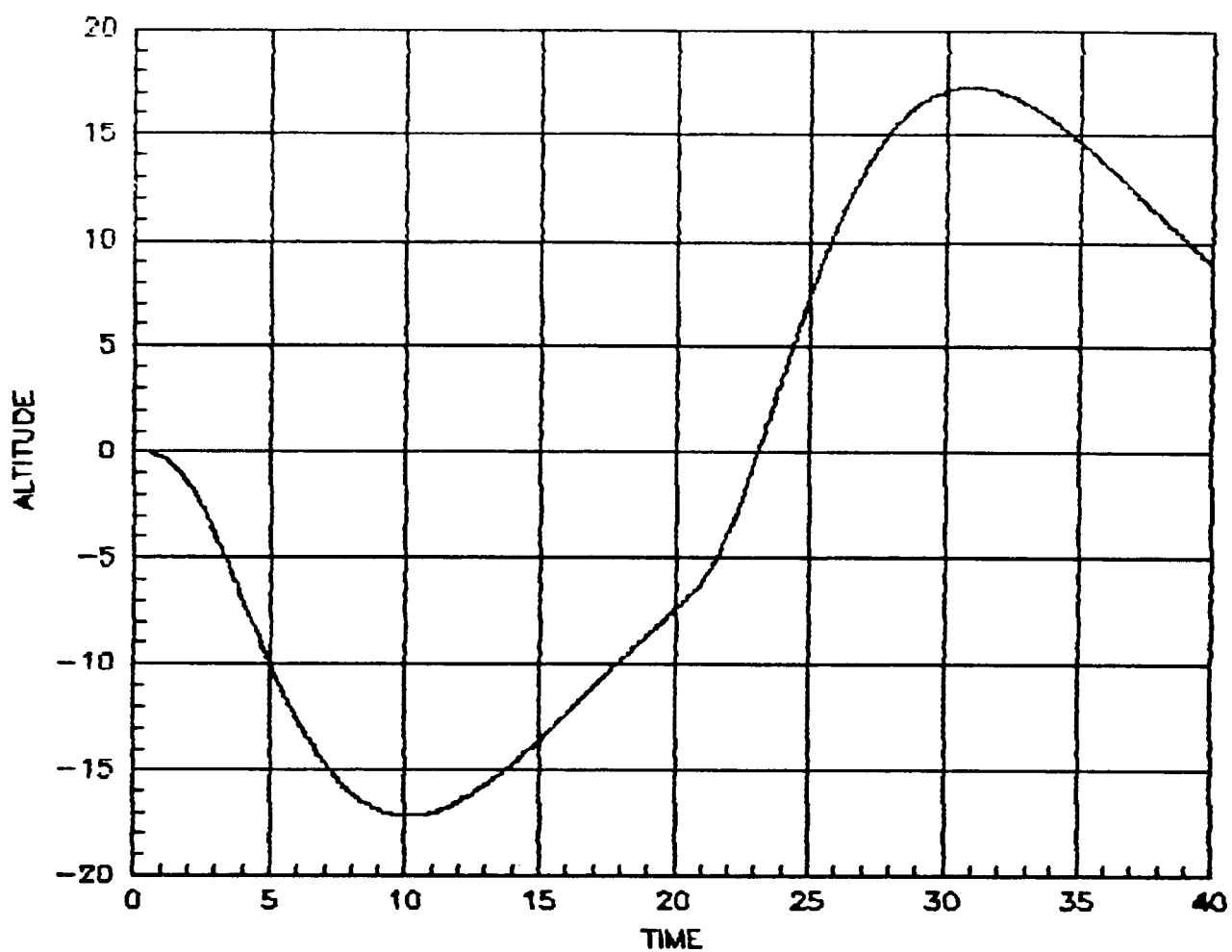
-5.9456D+01 +2.0293D+01i  
-5.9456D+01 -2.0293D+01i  
-1.3674D+02 -3.2629D-15i  
-4.8679D+01 +3.0434D-17i  
-6.7758D+01 +5.8218D+01i  
-6.7758D+01 -5.8218D+01i  
-1.0160D+02 +5.1723D-15i  
-1.0000D+02 +4.2144D-15i  
-8.8146D+01 +3.5469D-16i  
-2.3296D+01 +8.2436D-17i  
-7.4117D+00 +3.3574D+01i  
-7.4117D+00 -3.3574D+01i  
-7.9725D+00 +8.2655D+00i  
-7.9725D+00 -8.2655D+00i  
-1.2176D+01 +1.0192D+01i  
-1.2176D+01 -1.0192D+01i  
-2.9438D+00 +5.3113D+00i  
-2.9438D+00 -5.3113D+00i  
-2.3186D-01 +1.8504D-01i  
-2.3186D-01 -1.8504D-01i  
-3.4734D-01 -7.4748D-19i  
-6.5704D-02 -8.2699D-18i  
3.1435D-04 +4.3959D-03i  
3.1435D-04 -4.3959D-03i  
-5.4174D-01 +0.0000D+00i  
-5.0000D-01 +0.0000D+00i  
1.4997D-03 +0.0000D+00i  
-2.7600D-01 +0.0000D+00i  
-2.0000D-01 +0.0000D+00i  
-1.0000D+00 +0.0000D+00i  
-1.9200D+01 +0.0000D+00i

Closed-loop eigenvalues:

-1.3612D+02 +7.9936D-15i  
-1.1955D+02 +2.4338D-15i  
-1.0000D+02 +6.1189D-16i  
-8.8194D+01 +4.5058D-15i  
-8.8004D+01 -5.7886D+01i  
-8.8004D+01 +5.7886D+01i  
-6.1080D+01 +2.1706D+01i  
-6.1080D+01 -2.1706D+01i  
-4.9065D+01 +1.0959D-16i  
-2.6024D+01 +6.4031D-15i  
-1.9200D+01 +0.0000D+00i  
-5.4187D+00 +3.3073D+01i  
-5.4187D+00 -3.3073D+01i  
-5.0713D+00 -4.4306D+01i  
-5.0713D+00 +4.4306D+01i  
-4.1391D+00 -1.4054D+01i  
-4.1391D+00 +1.4054D+01i  
-3.2028D+00 +4.0631D+00i  
-3.2028D+00 -4.0631D+00i  
-1.0000D+00 +0.0000D+00i  
-6.9439D-01 +6.7315D-01i  
-6.9439D-01 -6.7315D-01i  
-5.0000D-01 +0.0000D+00i  
-4.7765D-01 +0.0000D+00i  
-3.4069D-01 +1.1513D-17i  
-2.7000D-01 +1.3002D-17i  
-2.5000D-01 +1.3092D-17i  
-2.0045D-01 -2.0037D-01i  
-2.0045D-01 +2.0037D-01i  
-1.5000D-01 -1.0000D-01i  
-1.5000D-01 +1.0000D-01i  
-1.4000D-01 -1.4000D-01i  
-1.4000D-01 +1.4000D-01i



CLOSED-LOOP TIME RESPONSE  
MACH NO VS TIME FOR A RAMP MACH NO COMMAND



CLOSED-LOOP TIME RESPONSE  
ALTITUDE VS TIME FOR A RAMP MACH NO COMMAND



APPENDIX A-III

AIRCRAFT - CAS LINEARIZED AND  
FTTC DESIGN USING EIGENSTRUCTURE ASSIGNMENT  
AT 0.7 MACH, 10000 ALTITUDE

## 1. F MATRIX (31 x 31)

[illegible][illegible]

## F Matrix (Cont'd)

COLUMNS 13 THRU 18

[illegible]

COLUMNS 19 THRU 24

[illegible]

## F Matrix (Cont'd)

COLUMNS 25 THRU 30

COLUMN 31

[illegible]

2. G-Matrix (31 x 4)

[illegible]

### 3. H-Matrix (17 x 31).

COLUMNS 1 THRU 6					
-3.4792D-09	9.0949D-10	0.0000D+00	0.0000D+00	-4.3656D+01	-2.8255D+00
2.9388D-03	3.1117D+01	5.8949D-05	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
7.6271D-04	-6.6234D+00	-3.1248D+00	1.0976D-03	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00
3.5246D-11	-6.3682D-09	0.0000D+00	0.0000D+00	-6.7738D+00	0.0000D+00
-1.3023D-10	-4.9207D-08	0.0000D+00	0.0000D+00	1.0573D+01	-4.8592D-02
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
9.2815D-04	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
2.9309D-03	3.1042D+01	2.9147D-05	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	-7.5418D+02	0.0000D+00	7.5418D+02	0.0000D+00	0.0000D+00
9.9943D-01	-2.5208D+01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.6793D-04	1.7293D+00	-2.1420D-08	5.5879D-06	0.0000D+00	0.0000D+00

COLUMNS 7 THRU 12					
1.5447D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	-5.1791D-06	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	6.0962D-06	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	-5.8208D-08	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-7.5853D-01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	9.7656D-01	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	-5.1863D-06	0.0000D+00	0.0000D+00
0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	1.1792D-02	0.0000D+00	0.0000D+00

## H-Matrix (Cont'd)

COLUMNS 13 THRU 18

[illegible]

COLUMNS 19 THRU 24

[illegible]

H-Matrix (Cont'd)

COLUMNS 25 THRU 30					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-2.1310D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-1.1973D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-5.1451D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00

COLUMN 31

0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00
0.0000D+00



## Controller Design

$$U = \text{Gain} * z$$

where

z are perturbed values of

$$[\phi \ p \ h \ \dot{h} \ a_x \ M \ fM \ fh]^T$$

and

$$u = [\delta T \ \delta e_{ap} \ \delta a_{ap} \ \delta \gamma_{ap}]^T$$

GAIN =

COLUMNS 1 THRU 6					
-1.4777D-02	-7.3096D-02	-9.2850D-02	-1.7141D-01	-1.3559D+02	-1.5352D+03
1.1924D-02	3.2934D-02	1.0155D-02	3.9151D-02	-2.0442D+00	-1.9619D+01
-2.5437D+00	-1.2855D+01	2.9255D-11	4.3619D-11	7.2545D-08	-5.4697D-06
1.4103D-01	-8.3187D-02	-5.8683D-12	2.9327D-11	-5.9810D-08	-7.4338D-07

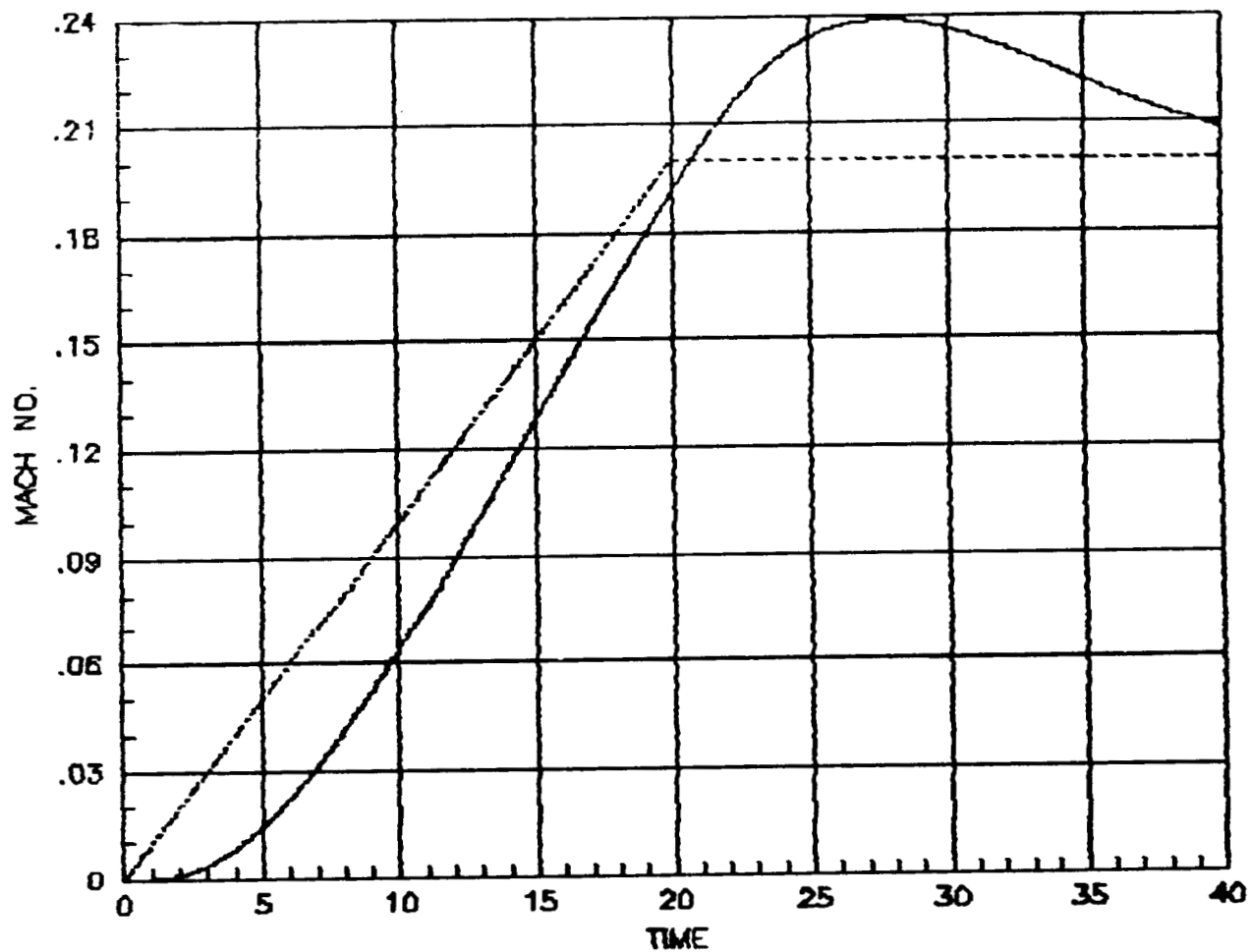
COLUMNS 7 THRU 8	
-1.3916D+02	-7.2697D-03
-2.4791D+00	1.0732D-03
-1.3702D-06	3.7197D-12
-2.3738D-08	-1.1493D-13

### Open-loop eigenvalues

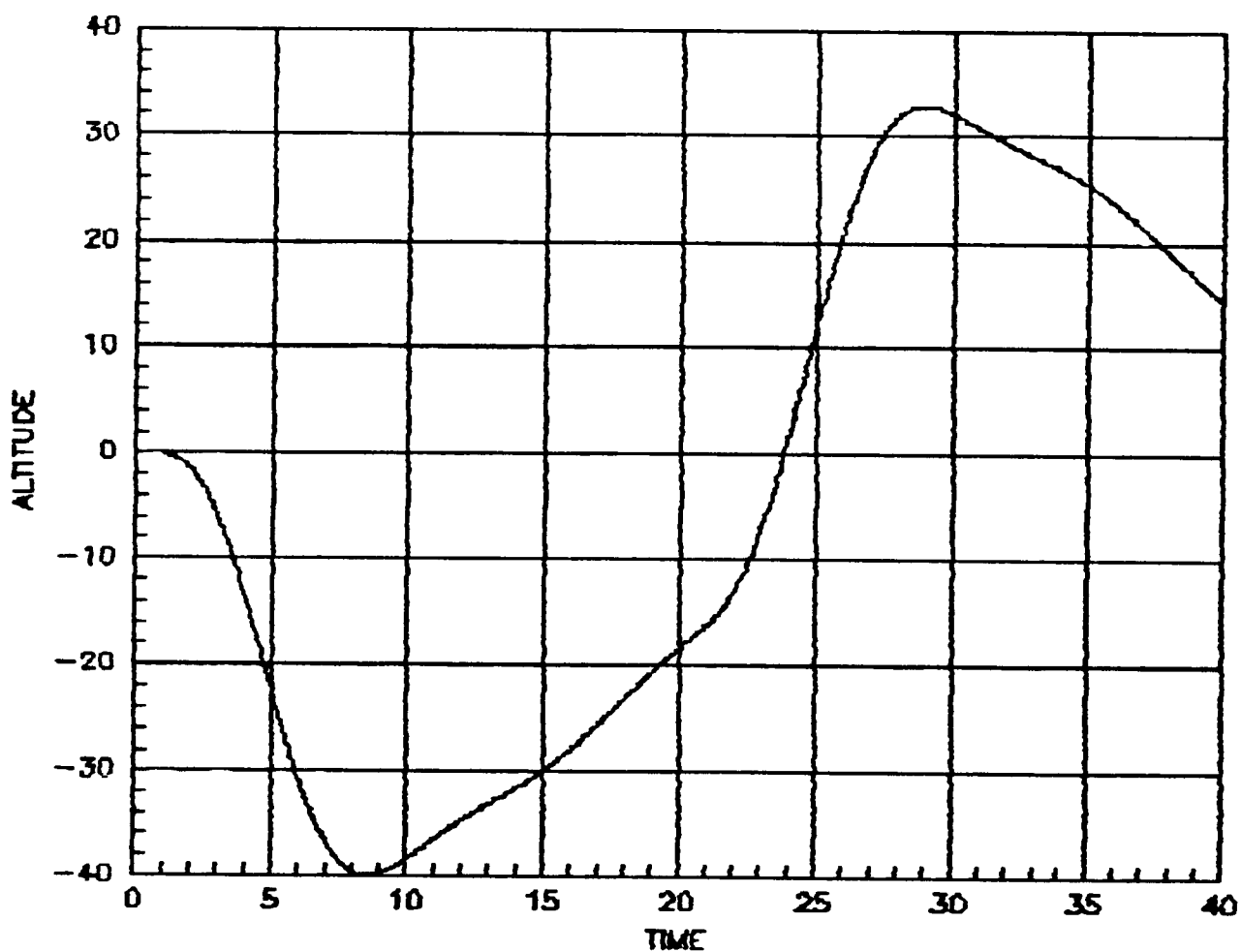
-1.2884D+02 -8.8818D-16i  
-8.5428D+01 -4.9351D+01i  
-8.5428D+01 +4.9351D+01i  
-8.8038D+01 +5.9819D-17i  
-1.0060D+02 +1.1917D-16i  
-5.0262D+01 -1.1145D-15i  
-8.8175D+01 +1.8883D-16i  
-1.0000D+02 +2.8730D-15i  
-2.8887D+01 -3.6097D+00i  
-2.8887D+01 +3.6097D+00i  
-1.4689D+01 -2.7807D+01i  
-1.4689D+01 +2.7807D+01i  
-1.5371D+01 -4.0164D-16i  
-3.8389D+00 -2.4781D+00i  
-3.8389D+00 +2.4781D+00i  
-7.8327D+00 +1.0559D-16i  
-1.7710D+00 +3.1453D+00i  
-1.7710D+00 -3.1453D+00i  
-4.8829D-01 +7.9997D-16i  
-6.1246D-01 +1.7102D-17i  
-2.7438D-01 +7.6776D-02i  
-2.7438D-01 -7.6776D-02i  
-5.0000D-01 -2.3109D-17i  
-1.2194D-02 +1.4478D-16i  
-5.2200D-11 +5.6029D-15i  
1.6208D-03 -5.8158D-15i  
7.6579D-03 +1.4094D-17i  
-1.7139D-01 -5.1627D-18i  
-2.0000D-01 +0.0000D+00i  
-1.0000D+00 +0.0000D+00i  
-1.9200D+01 +0.0000D+00i

### Closed-loop eigenvalues

-1.2883D+02 +0.0000D+00i  
-1.0264D+02 -1.6401D-15i  
-1.0000D+02 -4.2141D-16i  
-8.8174D+01 -2.6184D-15i  
-8.7871D+01 -2.8962D-15i  
-8.5441D+01 +4.9337D+01i  
-8.5441D+01 -4.9337D+01i  
-5.0270D+01 +6.0993D-16i  
-2.8984D+01 -3.6460D+00i  
-2.8984D+01 +3.6460D+00i  
-1.9200D+01 +0.0000D+00i  
-1.4759D+01 -2.7853D+01i  
-1.4759D+01 +2.7853D+01i  
-1.0067D+01 +1.2750D+01i  
-1.0067D+01 -1.2750D+01i  
-3.6721D+00 +1.6772D+00i  
-3.6721D+00 -1.6772D+00i  
-2.1011D+00 +3.1638D+00i  
-2.1011D+00 -3.1638D+00i  
-1.0000D+00 +0.0000D+00i  
-5.5401D-01 +0.0000D+00i  
-5.0000D-01 +0.0000D+00i  
-3.8063D-01 -1.9930D-17i  
-2.7000D-01 -1.1628D-17i  
-2.5000D-01 +1.8920D-17i  
-2.0023D-01 -2.0010D-01i  
-2.0023D-01 +2.0010D-01i  
-1.9303D-01 -7.2059D-01i  
-1.9303D-01 +7.2059D-01i  
-1.5000D-01 -1.0000D-01i  
-1.5000D-01 +1.0000D-01i  
-1.4000D-01 +1.4000D-01i  
-1.4000D-01 -1.4000D-01i



CLOSED-LOOP TIME RESPONSE  
MACH NO VS TIME FOR A RAMP MACH NO COMMAND



CLOSED-LOOP TIME RESPONSE  
ALTITUDE VS TIME FOR A RAMP MACH NO    COMMAND

APPENDIX A-IV

DESIRED EIGENVALUES AND EIGENVECTORS  
USED IN THE SYNTHESIS

# DESIRED EIGENVALUES:

```

-2.0000D-01 +2.0000D-01i
-2.0000D-01 -2.0000D-01i
-1.5000D-01 +1.0000D-01i
-1.5000D-01 -1.0000D-01i
-1.4000D-01 +1.4000D-01i
-2.5000D-01 +0.0000D+00i
-1.4000D-01 -1.4000D-01i
-2.7000D-01 +0.0000D+00i
    
```

# DESIRED EIGENVECTORS:

## MEASUREMENTS

$\epsilon$	p	h	h	$a_x$	M	$f_M$	$f_h$	
0.	0.	0.	0.	99.	99.	99.	0.	v
0.	0.	99.	99.	1.	99.	0.	99.	$\alpha$
0.	0.	99.	99.	0.	0.	0.	99.	q
0.	0.	0.	1.	99.	0.	99.	0.	$\theta$
0.	0.	0.	0.	0.	0.	0.	0.	$\beta$
99.	1.	0.	99.	0.	0.	0.	0.	p
0.	0.	0.	0.	0.	0.	0.	0.	r
1.	99.	99.	99.	0.	0.	0.	0.	$\phi$
0.	0.	1.	99.	99.	0.	99.	99.	h
0.	0.	0.	0.	99.	1.	0.	0.	Thrust Actuator
99.	99.	99.	99.	99.	99.	99.	99.	} States
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	} CAS States
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	99.	99.	99.	99.	99.	99.	
99.	99.	0.	0.	99.	99.	1.	0.	$f_M$
99.	99.	99.	99.	0.	0.	0.	1.	$f_h$

## APPENDIX B

### A DEMONSTRATION EXAMPLE FOR EXACT NONLINEAR CONTROLLER DESIGN WITH PRELINEARIZING TRANSFORMATION

## INTRODUCTION

The theory of prelinearizing transformations for a class of nonlinear feedback system design has already achieved maturity. Pioneering work on these transformations for flight control problems was carried out at NASA Ames research center by G. Meyer and others [1-14]. Though several papers are currently available, they deal with large dimension problems and consequently do not contain sufficient detail to serve as introductory example.

The objective of this technical memo is to illustrate the application of these transformations with a simple but realistic example. The theory is excluded completely from the discussions since the transformation scheme for this example turns out to be direct. The performance of the exact nonlinear controller is compared with a gain scheduled linear perturbation controller to bring out the advantages of the former in clear detail.

## ILLUSTRATIVE EXAMPLE:

Consider a two state variable model of a transport aircraft landing in the presence of winds as follows

$$\dot{V} = \frac{\eta T - D}{m V \cos \gamma} - \frac{g}{V} \tan \gamma \quad (1)$$

$$\dot{h} = \tan \gamma \quad (2)$$

here,  $V$  is airspeed,  $h$  altitude,  $T$  maximum thrust,  $D$  aerodynamic drag,  $\gamma$  flight-path angle, and  $\eta$  is the throttle setting.  $\gamma$  and  $\eta$  are the control variables in this model and down range is the independent variable. Now, a hypothetical thrust and drag characteristics are assumed as

$$T = T_0 (V + V_w)$$

$$D = e^{-\beta h} D_0 (V + V_w)^2$$



and  $V_w = V_w(h)$ ,  $V_w$  is the wind speed specified along flight path.

Note that the engine model is grossly incorrect at low speeds.

It is required that the aircraft follow a given trajectory  $V_R(t)$ ,  $h_R(t)$ .

Since there are two state variables and two control variables in this problem, consider a transformation of the form

$$x_1 = V, x_2 = h$$

$$u_1 = \frac{nT-D}{mV\cos\gamma} - \frac{g}{V} \tan\gamma; u_2 = \tan\gamma$$

such that the transformed system is the form

$$\dot{x}_1 = u_1 \tag{3}$$

$$\dot{x}_2 = u_2 \tag{4}$$

This model is in Brunovsky's canonical form according to Meyer [10]. If the number controls are less than the number of states, a simple transformation such as the one given above is no longer possible. A transformation is still feasible under certain weak conditions but will involve additional algebra.

To consider the regulator problem first, a controller of the form

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} V \\ h \end{bmatrix}$$

can be designed for the system (3), (4) using any of the available techniques for linear system design. Note, however that the designed controller should satisfy physical constraints on  $u_1$  and  $u_2$ .

Real controls can then be computed as

$$\gamma = \tan^{-1}(k_{21}V + k_{22}h) \quad (5)$$

$$\eta = [k_{11}V + k_{12}h + \frac{g}{V} \tan \gamma] \frac{mV \cos \gamma}{T} + \frac{D}{T} \quad (6)$$

Expressions (5) and (6) give the nonlinear regulator for the aircraft problem. A block diagram of the plant with regulator is given in Fig. 1.

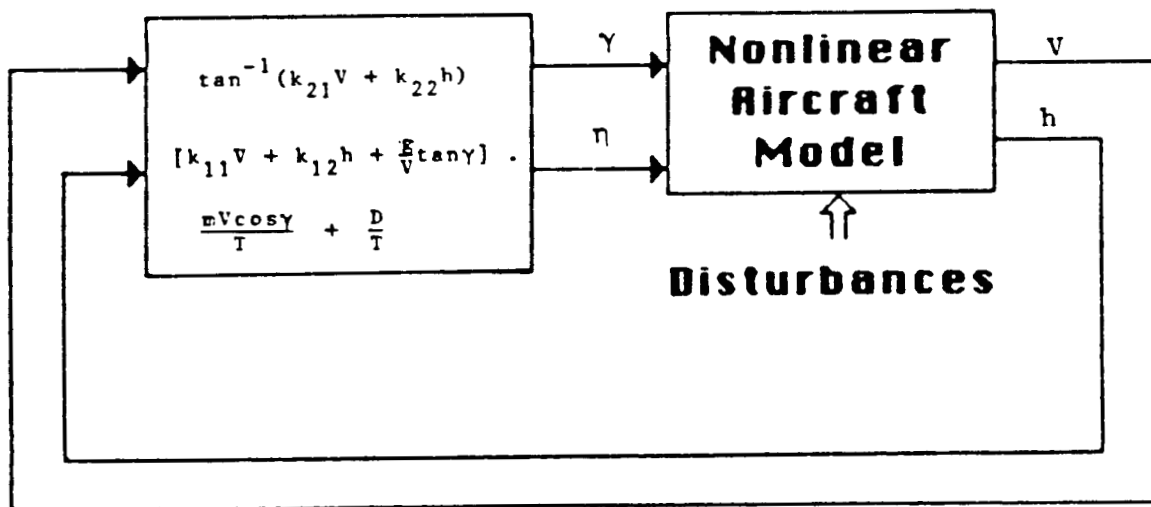


Figure 1. Nonlinear Regulator with Aircraft

The tracking problem is addressed next. Define the new state variables

$$\begin{aligned} x &= V - V_R \\ y &= h - h_R \end{aligned}$$

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OF POOR QUALITY

$$\dot{x} = \frac{\eta T - D}{mV \cos \gamma} - \frac{g}{V} \tan \gamma - C_1$$

$$\dot{y} = \tan \gamma - C_2$$

where

$$\dot{V}_R = C_1, \dot{h}_R = C_2$$

using the transformations

$$x_1 = x, x_2 = y$$

$$u_1 = \frac{\eta T - D}{mV \cos \gamma} - \frac{g}{V} \tan \gamma - C_1$$

$$u_2 = \tan \gamma - C_2$$

one has the following

$$\dot{x}_1 = u_1 \tag{7}$$

$$\dot{x}_2 = u_2 \tag{8}$$

which is in the Brunovsky's canonical form.

As before, a feedback regulator can be synthesized for (7), (8) as shown below using linear design approaches.

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} V - V_R \\ h - h_R \end{bmatrix}$$

This controller can be transformed to original coordinates to give:

$$\gamma = \tan^{-1} [k_{21}(V - V_R) + k_{22}(h - h_R) + C_2] \tag{9}$$

$$\eta = [k_{11}(V-V_R) + k_{12}(h-h_R) + C_1 + \frac{g}{V} \tan \gamma] \frac{mV \cos \gamma}{T} + \frac{D}{T} \quad (10)$$

Expressions (9) and (10) give the nonlinear controller for tracking the commands  $V_R(t)$  and  $h_R(t)$ . If a tighter control is desired, additional integral feedbacks can be incorporated with very little difficulty.

#### NONLINEAR CONTROLLER EVALUATION

To evaluate the nonlinear controller (9) and (10), a feedback gain matrix was computed first using linear quadratic regulator theory with the following state and control weighting matrices.

$$R_{xx} = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}, \quad R_{uu} = \begin{bmatrix} 5000 & 0 \\ 0 & 10^8 \end{bmatrix}.$$

These computations were carried out with the hypothetical data

$$\frac{T_0}{m} = 0.04025, \quad \frac{D_0}{m} = 0.0136238 \times 10^{-2}.$$

To serve as a standard for comparison, a gain scheduled linear perturbation controller was designed with the same data. Four design conditions were used which are given in Table 1.

TABLE 1. LINEARIZATION POINTS FOR THE ILLUSTRATIVE EXAMPLE

Range feet	$h_R$ , $\frac{h}{\text{feet}}$	$V_R$ , $\frac{V}{\text{feet/s}}$	$C_2$ , $\dot{h}$	$C_1$ , $\dot{V}$
0	10000	350	-0.04667	-0.000467
150000	3000	280	-0.06	-0.0016
200000	0.0	200	0.0	-0.02
205000	0.0	60	0.0	0.0

Note that the linearization points were on the commanded trajectory.

Two simulations are next setup. The first one implementing the nonlinear controller (9) and (10) and the second one using the gain scheduled linear perturbation controller. A disturbance in the form a wind shear was introduced in the simulations for additional realism, (fig. 2).

Figures 2-4 give the results. Note that the nonlinear controller has a slightly superior tracking performance. The real advantage of using this controller, however, is that it performs well using one set of gains throughout the operating region while the linear perturbation controller requires a gain schedule to achieve a comparable performance.

### CONCLUSIONS

A simple illustrative example was discussed in this technical memo to demonstrate the power of the prelinearization transformations in synthesizing exact nonlinear controllers. Comparisons were made against a gain scheduled linear perturbation controllers to show relative advantages.

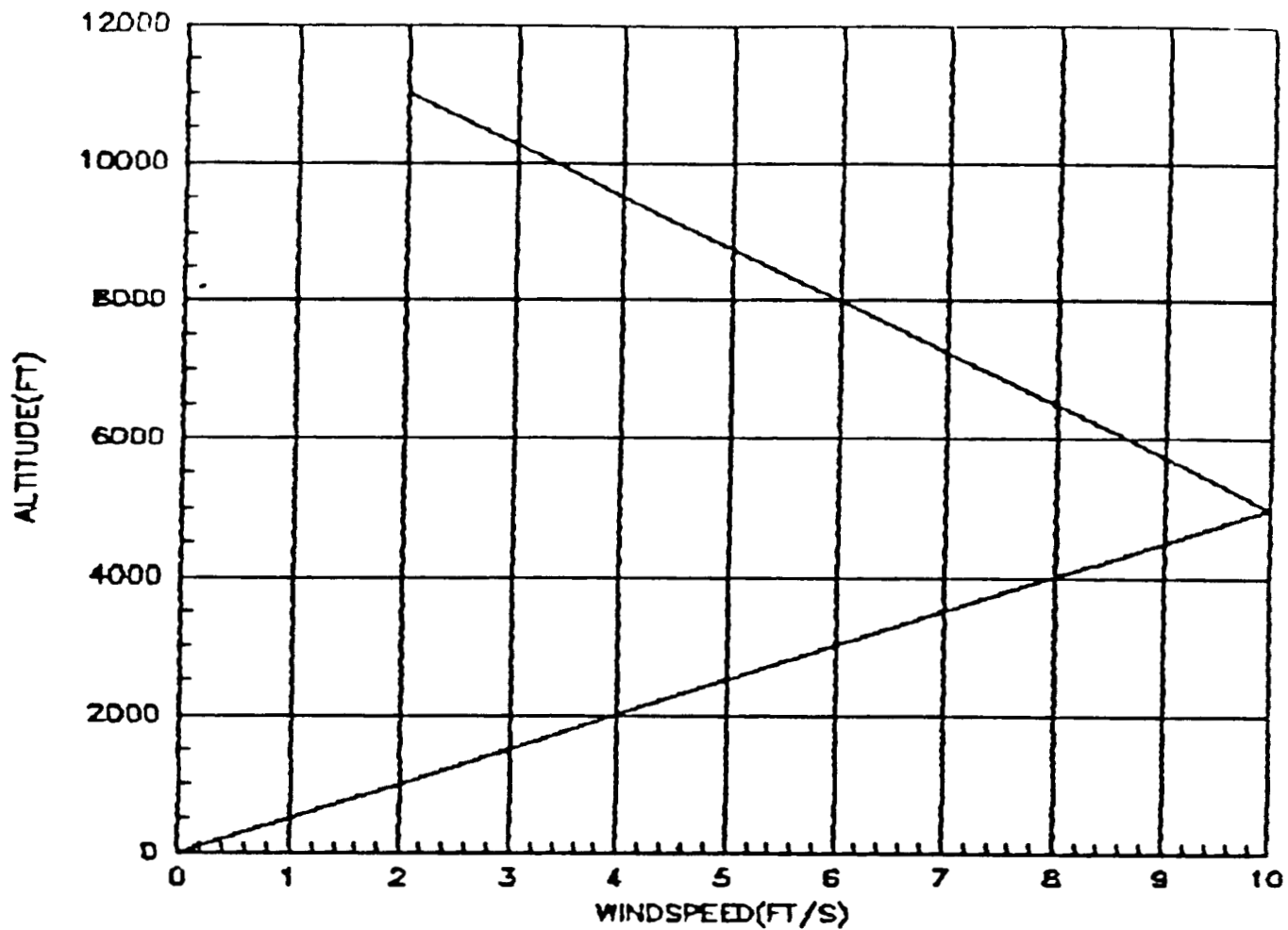


Figure 2. Wind Shear Used in Simulations

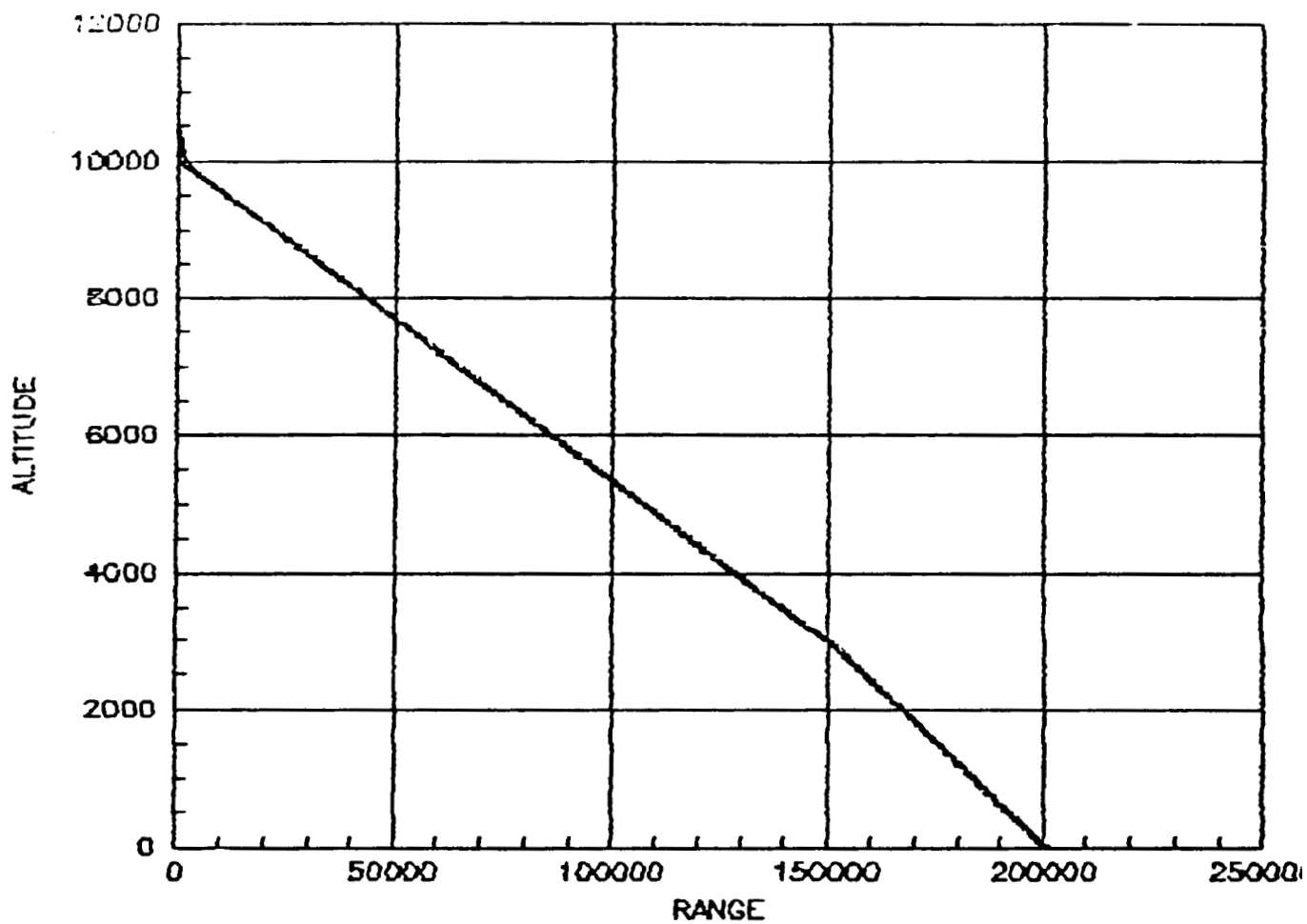


Figure 3. Comparison of the Performance of Linear and Nonlinear Controllers: Altitude vs. Range

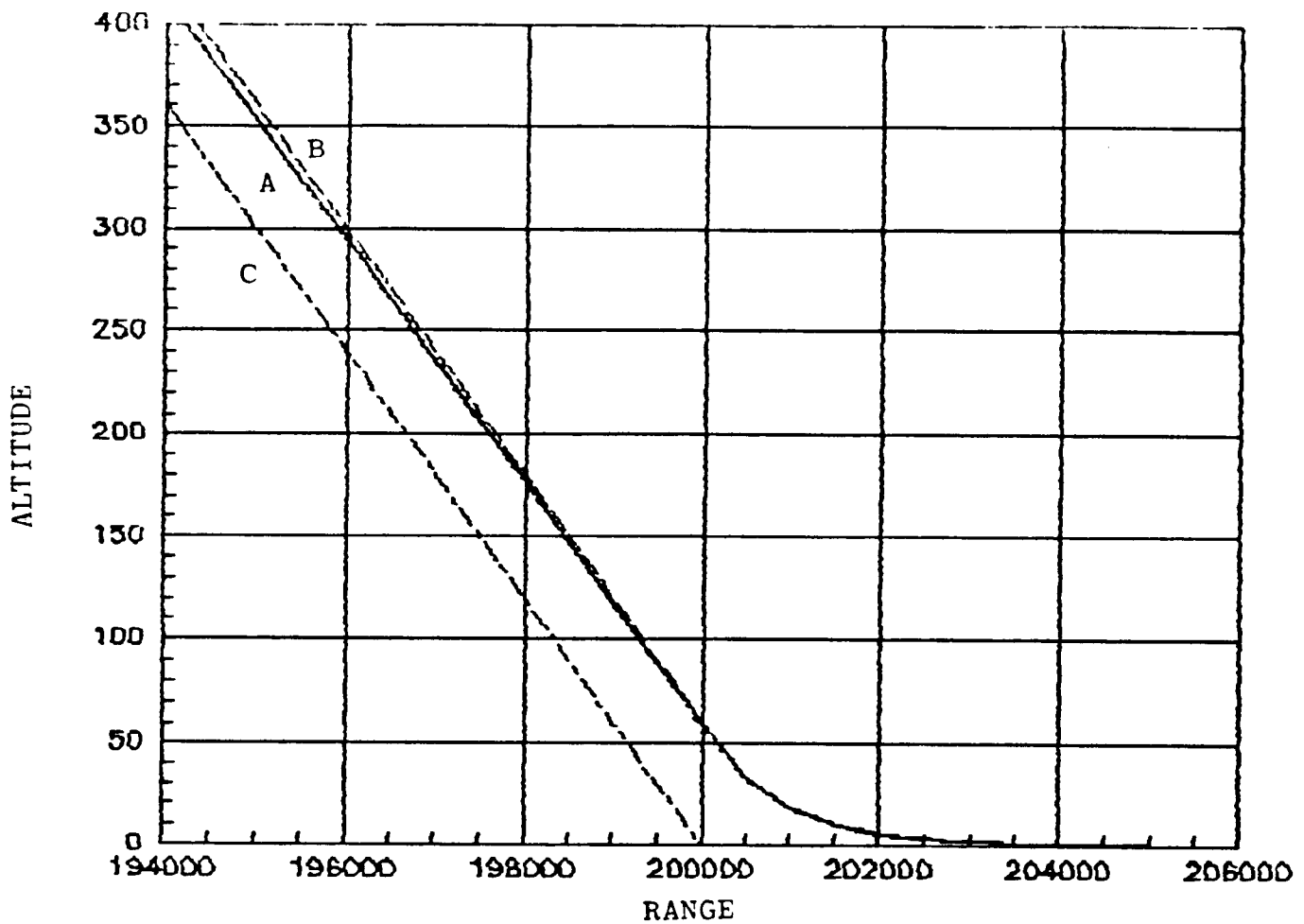


Figure 4. Comparison of the Performance of Linear and Nonlinear Controllers in the Vicinity of Touchdown: Altitude vs. Range

- A: Nonlinear Controller
- B: Linear Perturbation Controller
- C: Commanded Trajectory



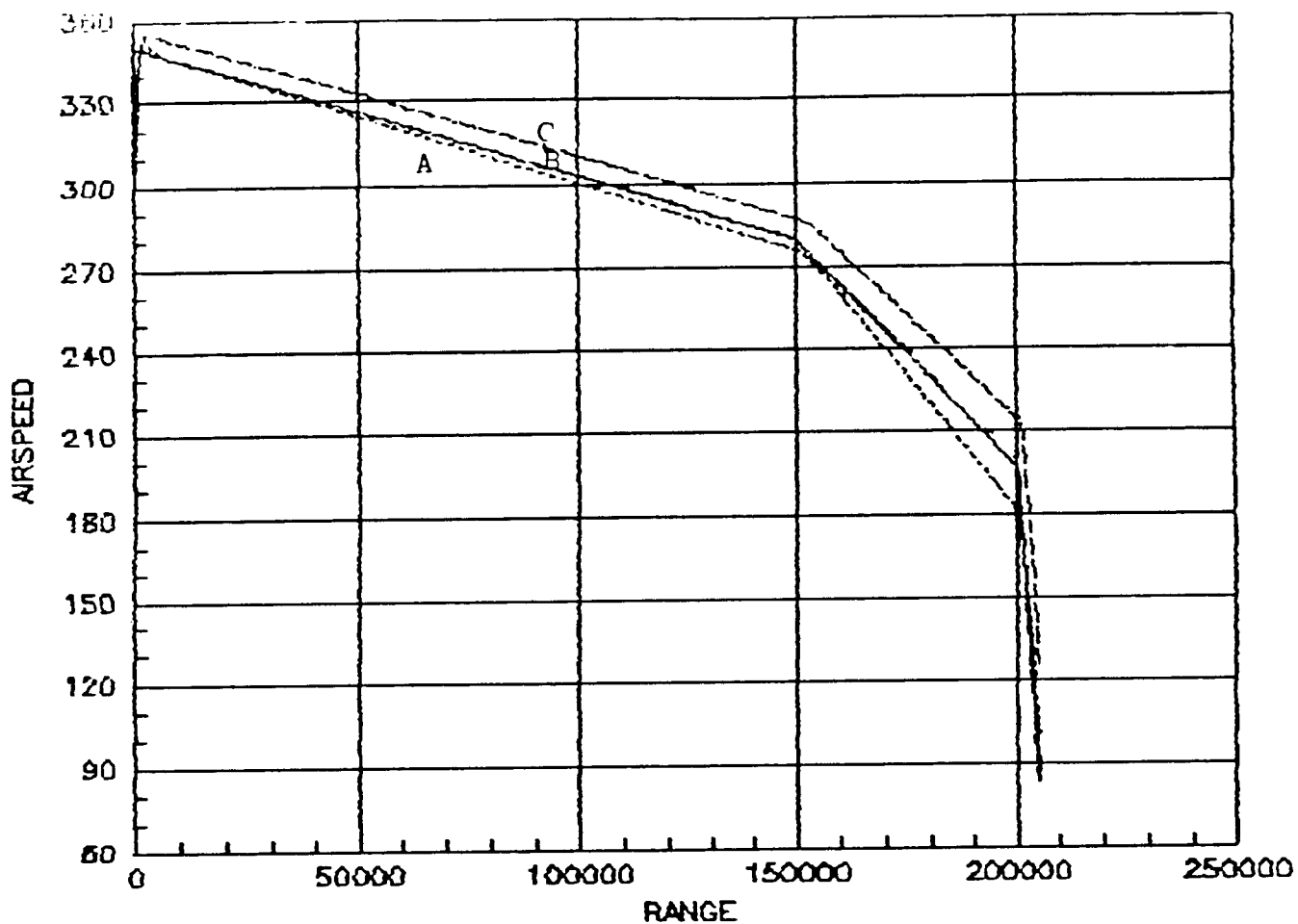


Figure 5. Comparison of the Performance of Linear and Nonlinear Controllers: Airspeed vs. Range

- A: Nonlinear Controller
- B: Commanded Trajectory
- C: Linear Perturbation Controller

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APPENDIX C

FORTRAN PROGRAM FOR MANEUVER MODELING

# COMMAND AND REFERENCE TRAJECTORY GENERATION ROUTINE FOR FLIGHT TEST TRAJECTORIES

COMMANDS FOR ALL FLIGHT TEST TRAJECTORIES ARE:

1. MACH NUMBER
2. ALTITUDE
3. ANGLE OF ATTACK
4. FLIGHT PATH ANGLE
5. ROLL ATTITUDE

REFERENCE TRAJECTORY CONSISTS OF

1. PITCH RATE  $\dot{q}$
2. YAW RATE  $\dot{r}$
3. ROLL RATE  $\dot{p}$
4. ANGLE OF SIDESLIP - BETA
5. THROTTLE
6. ELEVATOR
7. DIFFERENTIAL TAIL
8. RUDDER
9. AILERON

IN THE SYMMETRIC FLIGHT TEST TRAJECTORIES,  $q$  IS  
ASSUMED ZERO. CONSEQUENTLY, THETA IS A FAST VARIABLE.  
ALL OUTPUT ANGLES ARE IN RADIAN.

IOPT SPECIFIES THE MANEUVER

OPTIONS :

IOPT

0 : GENERATES TRANSIENTS FROM A GIVEN  
ALTITUDE-MACH PAIR TO ANOTHER  
ALTITUDE-MACH PAIR ; TIME OF FLIGHT  
SHOULD BE GIVEN

- INITIAL ALTITUDE=XI1
- INITIAL MACH =XI2
- FINAL ALTITUDE =XI3
- FINAL MACH =XI4
- TIME OF FLIGHT =XI5

1 : GENERATES LEVEL ACCELERATION/DECELERATION  
TRAJECTORY GIVEN INITIAL H.M; INTERMEDIATE H  
AND TIMES T1 AND T2

- INITIAL ALTITUDE=XI1
- INITIAL MACH =XI2
- FINAL MACH =XI3
- FINAL TIME =XI4

2 : GENERATES PUSH OVER/PULLUP TRAJECTORY GIVEN  
INITIAL H.M ; MINIMUM ALFA ; MAXIMUM  
ALFA AND TIMES T1, T2 AND T3

- INITIAL ALTITUDE =XI1
- INITIAL MACH =XI2
- MINIMUM ALFA =XI3
- MAXIMUM ALFA =XI4
- TIME FOR MIN. MACH=XI5
- TIME FOR MAX. MACH=XI6
- FINAL TIME =XI7

3 : GENERATES ZOOM AND PUSHOVER TRAJECTORY GIVEN  
APEX MACH NUMBER, ALTITUDE, ANGLE OF ATTACK  
AND THE MINIMUM H. ASSUMED HERE THAT THE  
TRAJECTORY BEGINS AND ENDS AT THE APEX SPEED  
AND MACH NUMBER. NOTE THAT THE APEX ALFA SHOULD

ORIGINAL PAGE IS  
OF POOR QUALITY

BE LESS THAN THE STRAIGHT AND LEVEL ALFA AT THE  
APEX ALTITUDE AND MACH NUMBER.

- APEX ALTITUDE =XI1
- APEX MACH =XI2
- APEX ALFA =XI3
- INITIAL ALTITUDE =XI4

4 : GENERATES THE EXCESS THRUST WINDUP TURN  
TRAJECTORY GIVEN THE INITIAL H.M, FINAL ALFA,  
AND THE TIME OF FLIGHT, ASSUMED THAT THE  
TIME TO RETURN THE AIRCRAFT FROM FINAL TURN  
TO LEVEL FLIGHT IS THE SAME AS TIME OF FLIGHT

- INITIAL ALTITUDE =XI1
- INITIAL MACH =XI2
- FINAL ALFA =XI3
- TIME OF FLIGHT =XI4
- DWELL TIME =XI5

5 : GENERATES THE CONSTANT THROTTLE WINDUP TURN  
TRAJECTORY GIVEN THE INITIAL H.M; FINAL ALFA,  
THE THROTTLE SETTING, MANEUVER TIME AND THE  
FINAL STABILIZATION TIME.

- INITIAL ALTITUDE =XI1
- INITIAL MACH =XI2
- INITIAL ALFA =XI3
- FINAL ALFA =XI4
- MANEUVER TIME =XI5
- FINAL/INITIAL  
STABILIZATION TIME=XI6
- THROTTLE SETTING =XI7

6 : GENERATES THE CONSTANT DYNAMIC PRESSURE AND  
CONSTANT LOAD FACTOR TRAJECTORY. GIVEN  
THE INITIAL H.M, FINAL MACH NUMBER, TIME OF  
FLIGHT AND THE REQUIRED DYNAMIC PRESSURE  
AND LOAD FACTOR. ASSUMED THAT THE FINAL  
TRANSIENT TIME IS THE SAME AS THE INITIAL  
TRANSIENT TIME, ALSO DESIRED DYNAMIC PRESSURE  
IS ASSUMED TO BE THE INITIAL DYNAMIC PRESSURE.

- INITIAL ALTITUDE =XI1
- INITIAL MACH =XI2
- FINAL MACH =XI3
- LOAD FACTOR =XI4
- TIME OF FLIGHT =XI5
- INITIAL TRANSIENT  
TIME =XI6

7 : GENERATES THE CONSTANT REYNOLD'S NUMBER AND  
CONSTANT LOAD FACTOR TRAJECTORY. GIVEN  
THE INITIAL H.M, FINAL MACH NUMBER, TIME OF  
FLIGHT AND THE REQUIRED REYNOLD'S NUMBER  
AND LOAD FACTOR. ASSUMED THAT THE DESIRED  
REYNOLD'S NUMBER IS THE INITIAL VALUE

- INITIAL ALTITUDE =XI1
- INITIAL MACH =XI2
- FINAL MACH =XI3
- LOAD FACTOR =XI4
- TIME OF FLIGHT =XI5
- INITIAL TRANSIENT  
TIME =XI6

IMPLICIT REAL\*8 (A-H,O-Z)  
DIMENSION TIME(1000), XMC(1000), HC(1000), ALPC(1000)  
DIMENSION GAMC(1000), PSI(1000), CRANCE(1000)



```

TIM=0. D0
DO 3 I=1. NPTS
XDOT=XDOT1
CALL TRIMS (H, XM, XSALE, ALF, BET, XLIFT, THRST, DRG, PHI,
1 THETA, P, Q, R, THRO, ELV, AIL, RUD, DTL, IER)
TIME (I) =TIM
FACL (I) =1. D0
XMC (I) =XM
HC (I) =H
CALL ATMO (H, ASP, RHO, XMU, DASPDH, DROOH, DMUDH)
DPRES (I) =0. 5D0*RHO*XM*XM*ASP*ASP
ALPC (I) =ALF*DR
GAMC (I) =0. D0
PHIC (I) =0. D0
PREF (I) =0. D0
QREF (I) =0. D0
RREF (I) =0. D0
BETREF (I) =0. D0
THROTTLE COMPUTATION
TR=(VDOT*XM*ASS+DRG)/DCOS (ALPC (I))
TMAX=THRST*100. D0/THRO
ETA=TR*100. D0/TMAX
IF (ETA. LT. 0. D0) ETA=0. D0
IF (ETA. GT. 100. D0) ETA=100. D0
THRO (I) =ETA
ELVR (I) =ELV
DTAILR (I) =DTL
RUDR (I) =RUD
AILR (I) =AIL
IF (TIM. LE. TE) GO TO 8
XDOT=0. D0
VDOT=0. D0
CONTINUE
XM=XM+XDOT*DT
TIME=TIME+DT
IF (TIM. GT. TE) XM=XMAX
CONTINUE
GO TO 8000
2000 IF (IOPT. GT. 2) GO TO 3000
COMMAND GENERATION FOR PUSHOVER/PULLUP TRAJECTORY
H=XI1
HI=H
CALL ATMO (H, ASP, RHO, XMU, DASPDH, DROOH, DMUDH)
XM=XI2
VEL=XM*ASP
ALPMIN=XI3*DR
ALPMAX=XI4*DR
TAMIN=XI5
TAMAX=XI6
TE=XI7
CALL TRIMS (H, XM, XSALE, ALF, BET, XLIFT, THRST, DRG, PHI,
1 THETA, P, Q, R, THRO, ELV, AIL, RUD, DTL, IER)
THET=THETA*DR
ALFA=ALF*DR
ALP0=ALFA
ALDOT1=(ALPMIN-ALP0)/TAMIN
ALDOT2=(ALPMAX-ALPMIN)/(TAMAX-TAMIN)
ALDOT3=(ALP0-ALPMAX)/(TE-TAMAX)
DT1=TAMIN/(0. 300*NTS)
DT2=(TAMAX-TAMIN)/(0. 300*NTS)

```

C C C

8

3

2000  
C C C C

```

DT3=(TE-TAMAX)/(0. 4D0*NTS)
DO 4 I=1. NPTS
ALDOT=ALDOT1
DT=DT1
CALL TRIMS (H, XM, XSALE, ALF, BET, XLIFT, THRST, DRG, PHI,
1 THETA, P, Q, R, THRO, ELV, AIL, RUD, DTL, IER)
TIME (I) =TIM
FACL (I) =1. D0
XMC (I) =XM
HC (I) =H
CALL ATMO (H, ASP, RHO, XMU, DASPDH, DROOH, DMUDH)
DPRES (I) =0. 5D0*RHO*XM*XM*ASP*ASP
ALPC (I) =ALFA
GAMC (I) =-(THET-ALPC (I))
PHIC (I) =0. D0
PREF (I) =0. D0
QREF (I) =0. D0
RREF (I) =0. D0
BETREF (I) =0. D0
THROTTLE COMPUTATION
LINEAR THROTTLE ASSUMPTION
TMAX=THRST*100. D0/THRO
TR=(G*DSIN (GAMC (I)) *XM*ASS+DRG)/DCOS (ALPC (I))
ETA=TR*100. D0/TMAX
IF (ETA. LT. 0. D0) ETA=0. D0
IF (ETA. GT. 100. D0) ETA=100. D0
THRO (I) =ETA
ELVR (I) =ELV
DTAILR (I) =DTL
RUDR (I) =RUD
AILR (I) =AIL
IF (TIM. GT. TAMIN) ALDOT=ALDOT2
IF (TIM. GT. TAMIN) DT=DT2
IF (TIM. GT. TAMAX) ALDOT=ALDOT3
IF (TIM. GT. TAMAX) DT=DT3
IF (TIM. GT. TE) ALDOT=0. D0
ALFA=ALFA+ALDOT*DT
HDOT=VEL*DSIN (GAMC (I))
H=H+HDOT*DT
CALL ATMO (H, ASP, RHO, XMU, DASPDH, DROOH, DMUDH)
VEL=XM*ASP
IF (TIM. LE. TE) GO TO 9
H=HI
CONTINUE
TIME=TIME+DT
CONTINUE
GO TO 8000
3000 IF (IOPT. GT. 3) GO TO 4000
COMMAND GENERATION FOR ZOOM AND PUSHOVER
TRAJECTORY
APEX QUANTITIES
HT=XI1
XMT=XI2
HE=HT
XFE=XMT
CALL ATMO (HT, ASP, RHO, XMU, DASPDH, DROOH, DMUDH)
VT=XMT*ASP
CALL TRIMS (HT, XMT, XSALE, ALF, BET, XLIFT, THRST, DRG, PHI,

```

C C C C C

9

4

3000  
C C C C

```

1  ALPF=ALF*DR
   ALPT=XI3*DR
   THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)

C  CDALFA AND CLALFA HAVE TO BE USED IN THE COMPUTATION OF TT
C  AND VERTICAL ACCELERATION IN THE FINAL VERSION.
C  TO BE CORRECTED
C  IN THE INITIAL PHASE, LIFT AT ZERO ANGLE OF ATTACK IS
C  ASSUMED TO BE ZERO.
C  DRAG IS ASSUMED TO BE THE LEVEL FLIGHT VALUE
C  TT=DRG/DCOS(ALPT)
C  TMAX=THRST*100.D0/THRO
C  TOT=TT*100.D0/TMAX
C  XLALEA=XLIET/ALPF
C  ALIFT=XLALEA*ALPT

C  VERTICAL ACCELERATION-ARTIFICIAL GRAVITY
C  CA=G-(TT*DSIN(ALPT)*ALIFT)/XMASS
C  E=HT*(VT*VT/(2.D0*GA))

C  QUANTITIES AT THE BEGINNING OF THE TEST PARABOLA
C  HI=XI4
C  VI=DSQRT(2.D0*GA*(E-HI))
C  CAMI=DACOS(VT/VI)
C  HDOTI=VI*DSIN(CAMI)
C  TIME OF FLIGHT FROM VI TO VT
C  DVT=DABS(VI*VI-VT*VT)
C  TI=DSQRT(DVT)/GA
C  TF1=2.D0*TI

C  INITIAL TRANSIENT PARABOLA
C  IPHASE=1

C  COMPUTE THE COEFFICIENTS OF THE ALTITUDE-TIME
C  POLYNOMIAL. TIME OF FLIGHT ASSUMED TO BE TF1
C  THIS CUBIC POLYNOMIAL IS USED TO GENERATE BOTH
C  INITIAL AND TERMINAL TRANSIENTS
C  A2=(HDOTI*TF1-2.D0*(HI-HT))/(TF1*TF1*TF1)
C  A1=(HDOTI-3.D0*A2*TF1*TF1)/(2.D0*TF1)

C  VDOT COMPUTATION : VDOT=CONSTANT IN THE INITIAL
C  AND TERMINAL TRANSIENTS
C  VDOT=(VI-VT)/TF1
C  VDOTI=VDOT

C  H=HT
C  XM=XMT
C  V=VT
C  GAMMA=0.D0
C  TIN=0.D0

C  TOTAL TIME OF FLIGHT AND STEP SIZE
C  TE=3.D0*TF1
C  TTE=2.D0*TF1
C  DT=TE/NITS
C  INDEXM=1
C  DO 5 I=1,NPTS
C  TIME(I)=TIM

```

```

EACL(I)=1.D0
XMC(I)=XM
HC(I)=H
CALL ATMO(H,ASP,RHO,XMU,DASDPH,DRODH,DMUDH)
DPRES(I)=0.5D0*RHO*XM*XM*ASP
CALL TRIMS(H,XM,XSALE,ALF,BET,XLIET,THRST,DRG,PHI,
1  THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)
C  ALA=ALF*DR
C  TMAX=THRST*100.D0/THRO
C  IF (IPHASE.EQ.2) GO TO 16
C  THROTTLE COMPUTATION FOR INITIAL AND TERMINAL
C  TRANSIENTS
C  TR=(VDOTI-C*DSIN(CAMC(I)))*XMASS*DRG/DCOS(ALPC(I))
C  ETA=TR*100.D0/TMAX
C  IF (ETA.LT.0.D0) ETA=0.D0
C  IF (ETA.GT.100.D0) ETA=100.D0
C  GO TO 17
C  CONTINUE
C  THROTTLE IS FIXED: COMPUTE ACTUAL THRUST
C  ETA=TOT
C  ACT=TMAX*TOT/100.D0
C  COMPUTE DLIFT/DALFA WITH THE ASSUMPTION THAT LIFT=0
C  AT ALFA=0
C  XLALEA=XLIET/ALA
C  ALA=(G-CA)*XMASS/((ACT*XLALEA)*DCOS(CAMMA))
C  CONTINUE
C  ALPC(I)=ALA
C  CAMC(I)=CAMMA
C  PHIC(I)=0.D0
C  PREF(I)=0.D0
C  QREF(I)=0.D0
C  RREF(I)=0.D0
C  BETREF(I)=0.D0
C  THRO(I)=ETA
C  ELVR(I)=ELV
C  DTALLR(I)=DTL
C  RUDR(I)=RUD
C  AILR(I)=AIL
C  TIME=TIN+DT
C  IF (TIM.GT.TF1) IPHASE=2
C  IF (TIM.GE.TTE) IPHASE=3
C  INITIAL TRANSIENT
C  IF (IPHASE.EQ.2 OR IPHASE.EQ.3) GO TO 11
C  H=HT+A1*TIM*TIM+A2*TIM*TIM*TIM
C  HDOT=2.D0*A1*TIM+3.D0*A2*TIM*TIM
C  VDOTI=VDOT
C  V=V+VDOTI*DT
C  CALL ATMO(H,ASP,RHO,XMU,DASDPH,DRODH,DMUDH)
C  XM=X/ASP
C  GAMMA=DASIN(HDOT/√)
C  INDEXM=INDEXM+1
C  GO TO 15
C  IF (IPHASE.EQ.3) GO TO 13
C  PARABOLIC TEST TRAJECTORY
C  DEL=TIM-TF1
C  IF (DEL.LT.0.D0) DEL=0.D0
C  H=HI+HDOTI*DEL-0.5D0*CA*DEL*DEL

```

```

13 HDOT=HDOTI-GA*DEL
C V=DSQRT(2.D0*GA*(E-H))
C ANGL=HDOT/V
C IF (ANGL.GT.1.D0) ANGL=1.D0
C IF (ANGL.LT.-1.D0) ANGL=-1.D0
C GAMMA=DASIN(ANGL)
C CALL ATMO(H,ASP,RHO,XMU,DASPDH,DRODH,DMUDH)
C XM=V/ASP
C GO TO 15
C IF (TIM.GT.TE) GO TO 14
C
C 14 TERMINAL TRANSIENT
C
C H=HC(INDEXM)
C GAMMA=GAMC(INDEXM)
C XM=XMC(INDEXM)
C VDOT1=-VDOT
C INDEXM=INDEXM-1
C IF (INDEXM.LT.1) INDEXM=1
C GO TO 15
C
C 15 TERMINAL STABILIZATION
C
C H=HE
C XM=XMT
C ALFA=ALPF
C VDOT1=0.D0
C GAMMA=0.D0
C CONTINUE
C CONTINUE
C GO TO 8000
C IF (IOP.T.GT.4) GO TO 5000
C
C 4000 COMMAND GENERATION FOR EXCESS THRUST WINDUP TURN
C TRAJECTORY
C
C H=XI1
C XM=XI2
C CALL TRIMS(H,XM,XSALF,ALF,BET,XLIFT,THRST,DRG,PHI,
C 1 THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)
C ALP=ALF
C ALPO=ALF
C ALPT=XI3
C ALFA IS COMPUTED IN DEGREES AND SUBSEQUENTLY CONVERTED
C TO RADIANS
C TF=XI4
C TD=XI5
C T1=TF
C T2=TF+TD
C T3=T2+TF
C ALDOT=(ALPT-ALP)/TF
C DT=(2.D0*TF+TD)/NTS
C TIM=0.D0
C DO 18 I=1,NPTS
C
C XLC=-1.D0
C TIME(I)=TIM
C XMC(I)=XM
C HC(I)=H
C CALL ATMO(H,ASP,RHO,XMU,DASPDH,DRODH,DMUDH)
C VEL=XM*ASP
C DPRES(I)=0.5D0*RHO*XM*ASP*ASP
C ALPC(I)=ALP*DR
C CALL TRIMS(H,XM,XLC,ALP,BET,XLIFT,THRST,DRG,PHI,
C 1 THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)
C
C
C
C

```

```

18 FACL(I)=XLC
C GAMC(I)=0.D0
C PHIC(I)=PHI*DR
C PREF(I)=P*DR
C RREF(I)=Q*DR
C RREF(I)=R*DR
C BETREF(I)=0.D0
C THRO(I)=THRO
C ELVR(I)=ELV
C DTAILR(I)=DTL
C RUDR(I)=RUD
C AILR(I)=AIL
C TIM=TIM*DT
C IF (TIM.LE.T1) ALP=ALP+ALDOT*DT
C IF (TIM.GT.T1 AND TIM.LE.T2) ALP=ALPT
C IF (TIM.GT.T2 AND TIM.LT.T3) ALP=ALP-ALDOT*DT
C IF (TIM.GT.T3) ALP=ALPO
C IF (ALP.LT.ALPO) ALP=ALPO
C CONTINUE
C GO TO 8000
C 5000 IF (IOP.T.GT.5) GO TO 6000
C
C 6000 COMMAND GENERATION FOR CONSTANT THROTTLE WINDUP TURN
C TRAJECTORY
C
C H=XI1
C XM=XI2
C CALL TRIMS(H,XM,XSALF,ALF,BET,XLIFT,THRST,DRG,PHI,
C 1 THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)
C ALPO=XI3
C ALPF=XI4
C TE=XI5
C TS=XI6
C TOT=XI7
C ALFA IS COMPUTED IN DEGREES AND SUBSEQUENTLY CONVERTED
C TO RADIANS
C ALD0=(ALPO-ALF)/TS
C ALD1=(ALPF-ALPO)/TF
C T1=TS
C T2=TS+TF
C T3=T2+TS
C ALP=ALF
C DT=T3/NTS
C TIM=0.D0
C ISW=0
C DO 19 I=1,NPTS
C
C XLC=-1.D0
C TIME(I)=TIM
C XMC(I)=XM
C HC(I)=H
C CALL ATMO(H,ASP,RHO,XMU,DASPDH,DRODH,DMUDH)
C VEL=XM*ASP
C DPRES(I)=0.5D0*RHO*VEL*VEL
C ALPC(I)=ALP*DR
C CALL TRIMS(H,XM,XLC,ALP,BET,XLIFT,THRST,DRG,PHI,
C 1 THETA,P,Q,R,THRO,ELV,AIL,RUD,DTL,IER)
C PHIC(I)=PHI*DR
C ETA=THRO
C GAMMA=0.D0
C
C 19 ACTUAL THRUST AND FLIGHT PATH ANGLE COMPUTATIONS
C IF (TIM.LT.T1 OR TIM.GE.T2) GO TO 20
C

```



[illegible][illegible]





INITIALIZATION  
IF (ISTART.GT.0) GO TO 100

DO 200 I=1,9  
DO 300 J=1,10  
DO 400 K=1,6  
LIFT(I,J,K)=WEIGHT  
DRAG(I,J,K)=0  
BETA(I,J,K)=0  
ALPHA(I,J,K)=0  
PHI(I,J,K)=0  
THETA(I,J,K)=0  
ROLLR(I,J,K)=0  
PITCHR(I,J,K)=0  
YAWR(I,J,K)=0  
THRUST(I,J,K)=0  
ELEVAT(I,J,K)=0  
DTAIL(I,J,K)=0  
RUDDER(I,J,K)=0  
THROTT(I,J,K)=0  
AILERO(I,J,K)=0

CONTINUE  
CONTINUE  
CONTINUE  
ISTART=1

STRAIGHT AND LEVEL/LEVEL TURN TRIMS USED AT EACH  
LOAD FACTOR ; LOAD FACTORS=ST. AND LEVEL, 2.4

	0.3	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
10K	*	*	*	*	*	*	*	*	*	*
15K	*	*	*	*	*	*	*	*	*	*
20K	*	*	*	*	*	*	*	*	*	*
25K	*	*	*	*	*	*	*	*	*	*
30K	*	*	*	*	*	*	*	*	*	*
35K	*	*	*	*	*	*	*	*	*	*
40K	*	*	*	*	*	*	*	*	*	*
45K	*	*	*	*	*	*	*	*	*	*
50K	*	*	*	*	*	*	*	*	*	*

DATA FROM THE LINEARIZATION PROGRAM

REFERENCE MACH NUMBERS

XMACH(1)=0.3  
XMACH(2)=0.4  
XMACH(3)=0.6  
XMACH(4)=0.8  
XMACH(5)=1.0  
XMACH(6)=1.2  
XMACH(7)=1.4  
XMACH(8)=1.6  
XMACH(9)=1.8  
XMACH(10)=2.0  
JMAX=10

REFERENCE ALTITUDES

ALT(1)=10000.  
ALT(2)=15000.  
ALT(3)=20000.  
ALT(4)=25000.  
ALT(5)=30000.  
ALT(6)=35000.  
ALT(7)=40000.

ALT(8)=45000.  
ALT(9)=50000.  
IMAX=9

REFERENCE LOAD FACTORS

XN(1)=1.D0  
XN(2)=2.D0  
XN(3)=4.D0  
KMAX=3

STRAIGHT AND LEVEL TRIMS AT LOAD FACTOR=1.  
ALT=10000

MACH=0.30000  
ALPHA(1,1,1)=11.43374  
THRUST(1,1,1)=6586.89893  
DRAG(1,1,1)=6456.14502  
ELEVAT(1,1,1)=0.10977  
THROTT(1,1,1)=13.72271

MACH=0.40000  
ALPHA(1,2,1)=6.32867  
THRUST(1,2,1)=3760.49414  
DRAG(1,2,1)=3737.57080  
ELEVAT(1,2,1)=0.06065  
THROTT(1,2,1)=7.83436

MACH=0.60000  
ALPHA(1,3,1)=2.74196  
THRUST(1,3,1)=4531.98633  
DRAG(1,3,1)=4526.80420  
ELEVAT(1,3,1)=0.02294  
THROTT(1,3,1)=9.44164

MACH=0.80000  
ALPHA(1,4,1)=1.38783  
THRUST(1,4,1)=7484.06982  
DRAG(1,4,1)=7481.85645  
ELEVAT(1,4,1)=0.00631  
THROTT(1,4,1)=15.59181

MACH=1.00000  
ALPHA(1,5,1)=1.04151  
THRUST(1,5,1)=19986.06445  
DRAG(1,5,1)=19982.78320  
ELEVAT(1,5,1)=0.01518  
THROTT(1,5,1)=41.63763

MACH=1.20000  
ALPHA(1,6,1)=0.68266  
THRUST(1,6,1)=43616.82813  
DRAG(1,6,1)=43613.79688  
ELEVAT(1,6,1)=0.02884  
THROTT(1,6,1)=90.86839

ALT(2)=15000.

MACH=0.30000  
ALPHA(2,1,1)=14.12306  
THRUST(2,1,1)=8196.08789  
DRAG(2,1,1)=7948.37109  
ELEVAT(2,1,1)=-0.14029

THROTT(2,1,1)=17.07518  
 MACH=0.40000  
 ALPHA(2,2,1)=7.76064  
 THRUST(2,2,1)=4492.39600  
 DRAG(2,2,1)=4451.17578  
 ELEVAT(2,2,1)=-0.07477  
 THROTT(2,1,1)=9.35916  
 MACH=0.60000  
 ALPHA(2,3,1)=3.33402  
 THRUST(2,3,1)=4014.46143  
 DRAG(2,3,1)=4007.66357  
 ELEVAT(2,3,1)=-0.02746  
 THROTT(2,3,1)=8.36346  
 MACH=0.80000  
 ALPHA(2,4,1)=1.69465  
 THRUST(2,4,1)=6128.50889  
 DRAG(2,4,1)=6126.50049  
 ELEVAT(2,4,1)=-0.00840  
 THROTT(2,4,1)=12.76793  
 MACH=1.00000  
 ALPHA(2,5,1)=1.22059  
 THRUST(2,5,1)=16509.15625  
 DRAG(2,5,1)=16504.97070  
 ELEVAT(2,5,1)=-0.01869  
 THROTT(2,5,1)=34.39407  
 MACH=1.20000  
 ALPHA(2,6,1)=0.85345  
 THRUST(2,6,1)=39445.27344  
 DRAG(2,6,1)=39440.91406  
 ELEVAT(2,6,1)=0.02145  
 THROTT(2,6,1)=82.17765  
 -----  
 ALT(3)=20000.  
 MACH=0.30000  
 ALPHA(3,1,1)=17.87607  
 THRUST(3,1,1)=10237.47266  
 DRAG(3,1,1)=9742.47266  
 ELEVAT(3,1,1)=-0.19544  
 THROTT(3,1,1)=21.32807  
 MACH=0.40000  
 ALPHA(3,2,1)=9.57196  
 THRUST(3,2,1)=5120.00635  
 DRAG(3,2,1)=5048.73633  
 ELEVAT(3,2,1)=-0.09187  
 THROTT(3,2,1)=10.66668  
 MACH=0.60000  
 ALPHA(3,3,1)=4.09133  
 THRUST(3,3,1)=3823.92310  
 DRAG(3,3,1)=3814.17993  
 ELEVAT(3,3,1)=0.03343  
 THROTT(3,3,1)=7.96651  
 MACH=0.80000  
 ALPHA(3,4,1)=2.08757  
 THRUST(3,4,1)=4988.27979  
 DRAG(3,4,1)=4984.80859

ELEVAT(3,4,1)=-0.01109  
 THROTT(3,4,1)=10.39225  
 MACH=1.00000  
 ALPHA(3,5,1)=1.45005  
 THRUST(3,5,1)=13561.25488  
 DRAG(3,5,1)=13556.93457  
 ELEVAT(3,5,1)=-0.02321  
 THROTT(3,5,1)=28.25261  
 MACH=1.20000  
 ALPHA(3,6,1)=1.07207  
 THRUST(3,6,1)=32175.04688  
 DRAG(3,6,1)=32169.41211  
 ELEVAT(3,6,1)=0.01202  
 THROTT(3,6,1)=67.03135  
 -----  
 ALT(4)=25000.  
 MACH=0.30000  
 ALPHA(4,1,1)=23.36872  
 THRUST(4,1,1)=13549.41016  
 DRAG(4,1,1)=12438.00781  
 ELEVAT(4,1,1)=-0.32833  
 THROTT(4,1,1)=28.22794  
 MACH=0.40000  
 ALPHA(4,2,1)=11.81394  
 THRUST(4,2,1)=6817.78898  
 DRAG(4,2,1)=6673.23438  
 ELEVAT(4,2,1)=-0.11645  
 THROTT(4,2,1)=14.20356  
 MACH=0.60000  
 ALPHA(4,3,1)=5.05511  
 THRUST(4,3,1)=3672.75806  
 DRAG(4,3,1)=3658.4646  
 ELEVAT(4,3,1)=-0.04267  
 THROTT(4,3,1)=7.65158  
 MACH=0.80000  
 ALPHA(4,4,1)=2.58838  
 THRUST(4,4,1)=4046.82544  
 DRAG(4,4,1)=4042.58105  
 ELEVAT(4,4,1)=-0.01454  
 THROTT(4,4,1)=8.43089  
 MACH=1.00000  
 ALPHA(4,5,1)=1.74225  
 THRUST(4,5,1)=11112.50000  
 DRAG(4,5,1)=11108.81348  
 ELEVAT(4,5,1)=0.02903  
 THROTT(4,5,1)=23.15104  
 MACH=1.20000  
 ALPHA(4,6,1)=1.34828  
 THRUST(4,6,1)=26115.28516  
 DRAG(4,6,1)=26108.00391  
 ELEVAT(4,6,1)=0.00017  
 THROTT(4,6,1)=54.40685  
 MACH=1.40000  
 ALPHA(4,7,1)=0.91263  
 THRUST(4,7,1)=34461.49219

DRAG(4,7,1)=34455.99219  
 ELEVAT(4,7,1)=0.03125  
 THROTT(4,7,1)=71.79477  
 ALT(5)=30000.  
 MACH=0.40000  
 ALPHA(5,2,1)=15.01142  
 THRUST(5,2,1)=8634.26074  
 DRAG(5,2,1)=8532.79199  
 ELEVAT(5,2,1)=0.14890  
 THROTT(5,2,1)=18.40471  
 MACH=0.60000  
 ALPHA(5,3,1)=6.29819  
 THRUST(5,3,1)=3736.00391  
 DRAG(5,3,1)=3713.16992  
 ELEVAT(5,3,1)=0.05459  
 THROTT(5,3,1)=7.76334  
 MACH=0.80000  
 ALPHA(5,4,1)=3.24018  
 THRUST(5,4,1)=3675.09839  
 DRAG(5,4,1)=3669.20386  
 ELEVAT(5,4,1)=0.01907  
 THROTT(5,4,1)=7.65646  
 MACH=1.00000  
 ALPHA(5,5,1)=2.12347  
 THRUST(5,5,1)=9070.81055  
 DRAG(5,5,1)=9064.57910  
 ELEVAT(5,5,1)=0.03670  
 THROTT(5,5,1)=18.89752  
 MACH=1.20000  
 ALPHA(5,6,1)=1.70491  
 THRUST(5,6,1)=21034.03711  
 DRAG(5,6,1)=21025.73242  
 ELEVAT(5,6,1)=0.01504  
 THROTT(5,6,1)=43.82091  
 MACH=1.40000  
 ALPHA(5,7,1)=1.20179  
 THRUST(5,7,1)=27754.65234  
 DRAG(5,7,1)=27747.45703  
 ELEVAT(5,7,1)=0.02093  
 THROTT(5,7,1)=57.82219  
 MACH=1.60000  
 ALPHA(5,8,1)=0.92773  
 THRUST(5,8,1)=34857.53125  
 DRAG(5,8,1)=34852.75781  
 ELEVAT(5,8,1)=0.03172  
 THROTT(5,8,1)=72.61986  
 ALT(6)=35000.  
 MACH=0.40000  
 ALPHA(6,2,1)=19.60772  
 THRUST(6,2,1)=11780.97754  
 DRAG(6,2,1)=11097.64063  
 ELEVAT(6,2,1)=0.22205  
 THROTT(6,2,1)=24.54370

MACH=0.60000  
 ALPHA(6,3,1)=7.88297  
 THRUST(6,3,1)=4551.47559  
 DRAG(6,3,1)=4508.51270  
 ELEVAT(6,3,1)=0.06980  
 THROTT(6,3,1)=9.48224  
 MACH=0.80000  
 ALPHA(6,4,1)=4.08973  
 THRUST(6,4,1)=3515.22437  
 DRAG(6,4,1)=3506.24780  
 ELEVAT(6,4,1)=0.02513  
 THROTT(6,4,1)=7.32338  
 MACH=1.00000  
 ALPHA(6,5,1)=2.62183  
 THRUST(6,5,1)=7394.08496  
 DRAG(6,5,1)=7386.25391  
 ELEVAT(6,5,1)=0.04689  
 THROTT(6,5,1)=15.40434  
 MACH=1.20000  
 ALPHA(6,6,1)=2.16504  
 THRUST(6,6,1)=16848.63867  
 DRAG(6,6,1)=16836.59570  
 ELEVAT(6,6,1)=0.03449  
 THROTT(6,6,1)=35.10133  
 MACH=1.40000  
 ALPHA(6,7,1)=1.57863  
 THRUST(6,7,1)=22229.19336  
 DRAG(6,7,1)=22218.51563  
 ELEVAT(6,7,1)=0.00749  
 THROTT(6,7,1)=46.31082  
 MACH=1.60000  
 ALPHA(6,8,1)=1.24392  
 THRUST(6,8,1)=27907.37891  
 DRAG(6,8,1)=27900.78516  
 ELEVAT(6,8,1)=0.02165  
 THROTT(6,8,1)=58.14037  
 MACH=1.80000  
 ALPHA(6,9,1)=0.92244  
 THRUST(6,9,1)=34814.69922  
 DRAG(6,9,1)=34809.92578  
 ELEVAT(6,9,1)=0.03189  
 THROTT(6,9,1)=72.53062  
 ALT(7)=40000.  
 MACH=0.60000  
 ALPHA(7,3,1)=10.07970  
 THRUST(7,3,1)=5843.59961  
 DRAG(7,3,1)=5753.18750  
 ELEVAT(7,3,1)=0.08769  
 THROTT(7,3,1)=12.17417  
 MACH=0.80000  
 ALPHA(7,4,1)=5.18231  
 THRUST(7,4,1)=3393.83130  
 DRAG(7,4,1)=3379.91748  
 ELEVAT(7,4,1)=0.03420  
 THROTT(7,4,1)=7.07048

C	C	THROTT(8,5,1)=12.85646
C	C	MACH=1.20000
		ALPHA(8,6,1)=3.49376
		THRUST(8,6,1)=11060.85254
		DRAG(8,6,1)=11040.61035
		ELEVAT(8,6,1)=-0.08991
		THROTT(8,6,1)=23.04344
C	C	MACH=1.40000
		ALPHA(8,7,1)=2.69042
		THRUST(8,7,1)=14250.15430
		DRAG(8,7,1)=14234.53125
		ELEVAT(8,7,1)=-0.03193
		THROTT(8,7,1)=29.68782
C	C	MACH=1.60000
		ALPHA(8,8,1)=2.17759
		THRUST(8,8,1)=17876.81055
		DRAG(8,8,1)=17864.27930
		ELEVAT(8,8,1)=-0.00825
		THROTT(8,8,1)=37.24335
C	C	MACH=1.80000
		ALPHA(8,9,1)=1.66031
		THRUST(8,9,1)=22117.89844
		DRAG(8,9,1)=22108.58789
		ELEVAT(8,9,1)=0.00834
		THROTT(8,9,1)=46.07896
C	C	MACH=2.00000
		ALPHA(8,10,1)=1.29021
		THRUST(8,10,1)=26857.76758
		DRAG(8,10,1)=26851.86477
		ELEVAT(8,10,1)=0.02017
		THROTT(8,10,1)=55.95368
C	C	ALT(9)=50000.
C	C	MACH=0.60000
		ALPHA(9,3,1)=17.14475
		THRUST(9,3,1)=10430.85449
		DRAG(9,3,1)=9967.41992
		ELEVAT(9,3,1)=-0.16728
		THROTT(9,3,1)=21.73095
C	C	MACH=0.80000
		ALPHA(9,4,1)=8.37713
		THRUST(9,4,1)=4699.29346
		DRAG(9,4,1)=4648.51465
		ELEVAT(9,4,1)=-0.06158
		THROTT(9,4,1)=9.79019
C	C	MACH=1.00000
		ALPHA(9,5,1)=5.21505
		THRUST(9,5,1)=5824.53369
		DRAG(9,5,1)=5800.21240
		ELEVAT(9,5,1)=-0.10767
		THROTT(9,5,1)=12.13445
C	C	MACH=1.20000
		ALPHA(9,6,1)=4.51515
		THRUST(9,6,1)=10089.23828
		DRAG(9,6,1)=10057.99707

C	C	MACH=1.00000
		ALPHA(7,5,1)=3.26914
		THRUST(7,5,1)=8617.98682
		DRAG(7,5,1)=8607.24072
		ELEVAT(7,5,1)=-0.06038
		THROTT(7,5,1)=13.78747
C	C	MACH=1.20000
		ALPHA(7,6,1)=2.75515
		THRUST(7,6,1)=13462.85840
		DRAG(7,6,1)=13447.48145
		ELEVAT(7,6,1)=-0.03916
		THROTT(7,6,1)=28.04762
C	C	MACH=1.40000
		ALPHA(7,7,1)=2.06871
		THRUST(7,7,1)=17758.31250
		DRAG(7,7,1)=17746.40039
		ELEVAT(7,7,1)=-0.00991
		THROTT(7,7,1)=36.99648
C	C	MACH=1.60000
		ALPHA(7,8,1)=1.65511
		THRUST(7,8,1)=22288.36523
		DRAG(7,8,1)=22278.38477
		ELEVAT(7,8,1)=0.00851
		THROTT(7,8,1)=46.43409
C	C	MACH=1.80000
		ALPHA(7,9,1)=1.24741
		THRUST(7,9,1)=27701.25391
		DRAG(7,9,1)=27694.62891
		ELEVAT(7,9,1)=0.02153
		THROTT(7,9,1)=57.71095
C	C	MACH=2.00000
		ALPHA(7,10,1)=0.95571
		THRUST(7,10,1)=33752.45313
		DRAG(7,10,1)=33747.78125
		ELEVAT(7,10,1)=0.03083
		THROTT(7,10,1)=70.31761
C	C	ALT(8)=45000.
C	C	MACH=0.60000
		ALPHA(8,3,1)=12.94587
		THRUST(8,3,1)=7824.69189
		DRAG(8,3,1)=7625.76953
		ELEVAT(8,3,1)=-0.11453
		THROTT(8,3,1)=16.30144
C	C	MACH=0.80000
		ALPHA(8,4,1)=6.55472
		THRUST(8,4,1)=4002.17065
		DRAG(8,4,1)=3976.03906
		ELEVAT(8,4,1)=-0.04578
		THROTT(8,4,1)=8.33786
C	C	MACH=1.00000
		ALPHA(8,5,1)=4.09879
		THRUST(8,5,1)=6171.10059
		DRAG(8,5,1)=6155.27441
		ELEVAT(8,5,1)=-0.07825

ELEVAT(9,6,1)=-0.13590  
THROTT(9,6,1)=21.01925

MACH=1.40000

ALPHA(9,7,1)=3.48176  
THRUST(9,7,1)=11492.67285  
DRAG(9,7,1)=11471.37109  
ELEVAT(9,7,1)=-0.05983  
THROTT(9,7,1)=23.94307

MACH=1.60000

ALPHA(9,8,1)=2.84358  
THRUST(9,8,1)=14410.39453  
DRAG(9,8,1)=14391.90820  
ELEVAT(9,8,1)=-0.02969  
THROTT(9,8,1)=30.02166

MACH=1.80000

ALPHA(9,9,1)=2.18668  
THRUST(9,9,1)=17727.35547  
DRAG(9,9,1)=17714.55469  
ELEVAT(9,9,1)=-0.00854  
THROTT(9,9,1)=36.93199

MACH=2.00000

ALPHA(9,10,1)=1.71658  
THRUST(9,10,1)=21436.30664  
DRAG(9,10,1)=21427.59766  
ELEVAT(9,10,1)=0.00654  
THROTT(9,10,1)=44.65897

SINCE LIFT DATA FOR SYMMETRIC FLIGHT HAS  
NOT BEEN STORED, COMPUTE IT  
FROM THRUST, ANGLE OF ATTACK AND WEIGHT. THETA=ALFA

DO 801 IN=1,IMAX  
DO 900 JN=1,JMAX  
ALRDAB=ALPHA(IN,JN,1)\*3.1415927D0/180.D0  
LIFT(IN,JN,1)=WEIGT\*THRUST(IN,JN,1)\*DSIN(ALRDAB)  
THETA(IN,JN,1)=ALPHA(IN,JN,1)  
CONTINUE  
CONTINUE

LEVEL TURN WHILE VARYING ALPHA

LOAD FACTOR=2.0

ALTITUDE=10000

MACH=0.4  
LIFT(1,2,2)=81322.34375  
DRAG(1,2,2)=16002.16016  
BETA(1,2,2)=0.20755  
ALPHA(1,2,2)=14.03178  
PHI(1,2,2)=62.29918  
THETA(1,2,2)=6.81347

ROLLR(1,2,2)=-0.93527  
PITCHR(1,2,2)=6.93060  
YAWR(1,2,2)=3.63877  
THRUST(1,2,2)=16494.32422  
ELEVAT(1,2,2)=-0.17681  
DTAIL(1,2,2)=-0.00683  
RUDDER(1,2,2)=-0.01629  
THROTT(1,2,2)=34.36317  
AILERO(1,2,2)=-0.02278

MACH=0.6

LIFT(1,3,2)=81324.18750  
DRAG(1,3,2)=7520.99805  
BETA(1,3,2)=0.07205  
ALPHA(1,3,2)=5.66266  
PHI(1,3,2)=60.42597  
THETA(1,3,2)=2.86451  
ROLLR(1,3,2)=-0.24961  
PITCHR(1,3,2)=4.38664  
YAWR(1,3,2)=2.46209  
THRUST(1,3,2)=7557.78125  
ELEVAT(1,3,2)=-0.05961  
DTAIL(1,3,2)=-0.00090  
RUDDER(1,3,2)=-0.00625  
THROTT(1,3,2)=15.74538  
AILERO(1,3,2)=-0.00300

MACH=0.8

LIFT(1,4,2)=81322.66406  
DRAG(1,4,2)=6050.01514  
BETA(1,4,2)=0.03316  
ALPHA(1,4,2)=2.88765  
PHI(1,4,2)=60.19688  
THETA(1,4,2)=1.46493  
ROLLR(1,4,2)=-0.09524  
PITCHR(1,4,2)=3.23173  
YAWR(1,4,2)=1.85107  
THRUST(1,4,2)=8060.24609  
ELEVAT(1,4,2)=-0.01999  
DTAIL(1,4,2)=-0.00038  
RUDDER(1,4,2)=-0.00327  
THROTT(1,4,2)=16.79218  
AILERO(1,4,2)=-0.00127

MACH=1.0

LIFT(1,5,2)=81317.89844  
DRAG(1,5,2)=21260.25391  
BETA(1,5,2)=0.01684  
ALPHA(1,5,2)=1.91904  
PHI(1,5,2)=60.30110  
THETA(1,5,2)=0.96567  
ROLLR(1,5,2)=-0.05048  
PITCHR(1,5,2)=2.60144  
YAWR(1,5,2)=1.48377  
THRUST(1,5,2)=21272.15820  
ELEVAT(1,5,2)=-0.03307  
DTAIL(1,5,2)=-0.00026  
RUDDER(1,5,2)=-0.00215  
THROTT(1,5,2)=44.31699  
AILERO(1,5,2)=-0.00088

MACH=1.2

LIFT(1,6,2)=81322.88281  
DRAG(1,6,2)=44835.46484



BETA(1,6,2)=0.00804  
 ALPHA(1,6,2)=1.52935  
 PHI(1,6,2)=60.48763  
 THETA(1,6,2)=0.76051  
 ROLL(1,6,2)=0.03339  
 PITCH(1,6,2)=2.18899  
 YAW(1,6,2)=1.23910  
 THRUST(1,6,2)=44851.43750  
 ELEVAT(1,6,2)=0.00820  
 DTAIL(1,6,2)=0.00018  
 RUDDER(1,6,2)=0.00167  
 THROTT(1,6,2)=93.44050  
 AILERO(1,6,2)=0.00061

ALTITUDE=15000

MACH=0.4  
 LIFT(2,2,2)=81281.48438  
 DRAG(2,2,2)=20832.15430  
 BETA(2,2,2)=0.23665  
 ALPHA(2,2,2)=18.27243  
 PHI(2,2,2)=63.80337  
 THETA(2,2,2)=8.51193  
 ROLL(2,2,2)=1.24015  
 PITCH(2,2,2)=7.43510  
 YAW(2,2,2)=3.65798  
 THRUST(2,2,2)=21940.05273  
 ELEVAT(2,2,2)=0.23985  
 DTAIL(2,2,2)=0.00863  
 RUDDER(2,2,2)=0.02101  
 THROTT(2,2,2)=45.70844  
 AILERO(2,2,2)=0.02877

MACH=0.5  
 LIFT(2,3,2)=81284.10938  
 DRAG(2,3,2)=8269.13281  
 BETA(2,3,2)=0.07800  
 ALPHA(2,3,2)=6.85644  
 PHI(2,3,2)=60.58232  
 THETA(2,3,2)=3.44813  
 ROLL(2,3,2)=0.30719  
 PITCH(2,3,2)=4.44093  
 YAW(2,3,2)=2.50414  
 THRUST(2,3,2)=8328.68945  
 ELEVAT(2,3,2)=0.07164  
 DTAIL(2,3,2)=0.00105  
 RUDDER(2,3,2)=0.00671  
 THROTT(2,3,2)=17.35144  
 AILERO(2,3,2)=0.00349

MACH=0.8  
 LIFT(2,4,2)=81295.99219  
 DRAG(2,4,2)=7632.95557  
 BETA(2,4,2)=0.03490  
 ALPHA(2,4,2)=3.51073  
 PHI(2,4,2)=60.23729  
 THETA(2,4,2)=1.77472  
 ROLL(2,4,2)=0.11763  
 PITCH(2,4,2)=3.29561  
 YAW(2,4,2)=1.88457  
 THRUST(2,4,2)=7646.93066  
 ELEVAT(2,4,2)=0.02459  
 DTAIL(2,4,2)=0.00041  
 RUDDER(2,4,2)=0.00346

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THROTT(2,4,2)=15.93111  
 AILERO(2,4,2)=0.00136

MACH=1.0  
 LIFT(2,5,2)=81285.80469  
 DRAG(2,5,2)=17773.39844  
 BETA(2,5,2)=0.01764  
 ALPHA(2,5,2)=2.28268  
 PHI(2,5,2)=60.30553  
 THETA(2,5,2)=1.14657  
 ROLL(2,5,2)=0.06104  
 PITCH(2,5,2)=2.64932  
 YAW(2,5,2)=1.51080  
 THRUST(2,5,2)=17787.41406  
 ELEVAT(2,5,2)=0.04055  
 DTAIL(2,5,2)=0.00029  
 RUDDER(2,5,2)=0.00225  
 THROTT(2,5,2)=37.05711  
 AILERO(2,5,2)=0.00095

ALTITUDE=20000

MACH=0.6  
 LIFT(3,3,2)=81246.71875  
 DRAG(3,3,2)=9649.65039  
 BETA(3,3,2)=0.08615  
 ALPHA(3,3,2)=8.41573  
 PHI(3,3,2)=60.84391  
 THETA(3,3,2)=4.19833  
 ROLL(3,3,2)=0.38376  
 PITCH(3,3,2)=4.56550  
 YAW(3,3,2)=2.54698  
 THRUST(3,3,2)=9754.68652  
 ELEVAT(3,3,2)=0.08665  
 DTAIL(3,3,2)=0.00131  
 RUDDER(3,3,2)=0.00732  
 THROTT(3,3,2)=20.32226  
 AILERO(3,3,2)=0.00438

MACH=0.8  
 LIFT(3,4,2)=81247.26563  
 DRAG(3,4,2)=7292.51221  
 BETA(3,4,2)=0.03716  
 ALPHA(3,4,2)=4.30461  
 PHI(3,4,2)=60.29356  
 THETA(3,4,2)=2.16853  
 ROLL(3,4,2)=0.14667  
 PITCH(3,4,2)=3.36444  
 YAW(3,4,2)=1.91955  
 THRUST(3,4,2)=7312.87744  
 ELEVAT(3,4,2)=0.03082  
 DTAIL(3,4,2)=0.00045  
 RUDDER(3,4,2)=0.00369  
 THROTT(3,4,2)=15.23516  
 AILERO(3,4,2)=0.00150

MACH=1.0  
 LIFT(3,5,2)=81245.67969  
 DRAG(3,5,2)=14766.45898  
 BETA(3,5,2)=0.01856  
 ALPHA(3,5,2)=2.74857  
 PHI(3,5,2)=60.31479  
 THETA(3,5,2)=1.37810  
 ROLL(3,5,2)=0.07477

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PITCH(3.5,2)=2.70026  
 YAWR(3.5,2)=1.53928  
 THRUST(3.5,2)=14783.45508  
 ELEVAT(3.5,2)=-0.05024  
 DTAIL(3.5,2)=-0.00031  
 RUDDER(3.5,2)=-0.00237  
 THROTT(3.5,2)=30.79886  
 AILERO(3.5,2)=-0.00104

MACH=1.2  
 LIFT(3.6,2)=81246.67188  
 DRAG(3.6,2)=33376.49609  
 BETA(3.6,2)=0.00903  
 ALPHA(3.6,2)=2.30240  
 PHI(3.6,2)=60.55615  
 THETA(3.6,2)=1.14012  
 ROLL(3.6,2)=-0.05208  
 PITCH(3.6,2)=2.27888  
 YAWR(3.6,2)=1.28638  
 THRUST(3.6,2)=33403.44922  
 ELEVAT(3.6,2)=-0.04116  
 DTAIL(3.6,2)=-0.00025  
 RUDDER(3.6,2)=-0.00182  
 THROTT(3.6,2)=69.59052  
 AILERO(3.6,2)=-0.00085

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 ALTITUDE=25000

MACH=0.6  
 LIFT(4.3,2)=81199.57813  
 DRAG(4.3,2)=12575.88477  
 BETA(4.3,2)=0.10038  
 ALPHA(4.3,2)=10.57505  
 PHI(4.3,2)=61.35437  
 THETA(4.3,2)=5.20321  
 ROLL(4.3,2)=-0.49201  
 PITCH(4.3,2)=4.74157  
 YAWR(4.3,2)=2.59009  
 THRUST(4.3,2)=12793.17871  
 ELEVAT(4.3,2)=-0.10793  
 DTAIL(4.3,2)=-0.00213  
 RUDDER(4.3,2)=-0.00831  
 THROTT(4.3,2)=26.65246  
 AILERO(4.3,2)=-0.00711

MACH=0.8  
 LIFT(4.4,2)=81200.09375  
 DRAG(4.4,2)=7021.35449  
 BETA(4.4,2)=0.04045  
 ALPHA(4.4,2)=5.30818  
 PHI(4.4,2)=60.37205  
 THETA(4.4,2)=2.66510  
 ROLL(4.4,2)=-0.18415  
 PITCH(4.4,2)=3.43894  
 YAWR(4.4,2)=1.95581  
 THRUST(4.4,2)=7052.07275  
 ELEVAT(4.4,2)=-0.03949  
 DTAIL(4.4,2)=-0.00052  
 RUDDER(4.4,2)=-0.00397  
 THROTT(4.4,2)=14.69182  
 AILERO(4.4,2)=-0.00172

MACH=1.0  
 LIFT(4.5,2)=81205.17969

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DRAG(4.5,2)=13663.21680  
 BETA(4.5,2)=0.01960  
 ALPHA(4.5,2)=3.34309  
 PHI(4.5,2)=60.36426  
 THETA(4.5,2)=1.67157  
 ROLL(4.5,2)=-0.09264  
 PITCH(4.5,2)=2.75925  
 YAWR(4.5,2)=1.56975  
 THRUST(4.5,2)=13686.49902  
 ELEVAT(4.5,2)=-0.06282  
 DTAIL(4.5,2)=-0.00035  
 RUDDER(4.5,2)=-0.00250  
 THROTT(4.5,2)=28.51354  
 AILERO(4.5,2)=-0.00116

MACH=1.2  
 LIFT(4.6,2)=81204.76563  
 DRAG(4.6,2)=27308.66602  
 BETA(4.6,2)=0.00967  
 ALPHA(4.6,2)=2.84451  
 PHI(4.6,2)=60.57278  
 THETA(4.6,2)=1.40686  
 ROLL(4.6,2)=-0.06556  
 YAWR(4.6,2)=1.31163  
 THRUST(4.6,2)=27342.36719  
 ELEVAT(4.6,2)=-0.06393  
 DTAIL(4.6,2)=-0.00031  
 RUDDER(4.6,2)=-0.00191  
 THROTT(4.6,2)=56.96326  
 AILERO(4.6,2)=-0.00102

MACH=1.4  
 LIFT(4.7,2)=81204.61719  
 DRAG(4.7,2)=36057.69922  
 BETA(4.7,2)=0.00587  
 ALPHA(4.7,2)=2.14089  
 PHI(4.7,2)=60.55597  
 THETA(4.7,2)=1.05789  
 ROLL(4.7,2)=-0.04225  
 PITCH(4.7,2)=1.99266  
 YAWR(4.7,2)=1.12482  
 THRUST(4.7,2)=36083.16406  
 ELEVAT(4.7,2)=-0.01267  
 DTAIL(4.7,2)=-0.00027  
 RUDDER(4.7,2)=-0.00156  
 THROTT(4.7,2)=75.17326  
 AILERO(4.7,2)=-0.00089

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 ALTITUDE=30000

MACH=0.6  
 LIFT(5.3,2)=81168.92969  
 DRAG(5.3,2)=17106.69336  
 BETA(5.3,2)=0.12090  
 ALPHA(5.3,2)=13.74951  
 PHI(5.3,2)=62.32314  
 THETA(5.3,2)=6.59289  
 ROLL(5.3,2)=-0.65416  
 PITCH(5.3,2)=5.01229  
 YAWR(5.3,2)=2.62893  
 THRUST(5.3,2)=17611.40430  
 ELEVAT(5.3,2)=-0.13957  
 DTAIL(5.3,2)=-0.00351  
 RUDDER(5.3,2)=-0.00966

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THROTT(5,3,2)=36.69043  
 AILERO(5,3,2)=-0.01159  
 C C  
 MACH=0.8  
 LIFT(5,4,2)=81162.42969  
 DRAG(5,4,2)=8255.35938  
 BETA(5,4,2)=0.04465  
 ALPHA(5,4,2)=6.61738  
 PHI(5,4,2)=60.55319  
 THETA(5,4,2)=3.30323  
 ROLL(5,4,2)=-0.23412  
 PITCH(5,4,2)=3.53236  
 YAWR(5,4,2)=1.99418  
 THRUST(5,4,2)=8310.72852  
 ELEVAT(5,4,2)=-0.05094  
 DTAIL(5,4,2)=-0.00060  
 RUDDER(5,4,2)=-0.00432  
 THROTT(5,4,2)=17.31402  
 AILERO(5,4,2)=-0.00200  
 C C  
 MACH=1.00000  
 LIFT(5,5,2)=81172.07813  
 DRAG(5,5,2)=12753.33301  
 BETA(5,5,2)=0.02089  
 ALPHA(5,5,2)=4.13008  
 PHI(5,5,2)=60.43553  
 THETA(5,5,2)=2.05866  
 ROLL(5,5,2)=-0.11670  
 PITCH(5,5,2)=2.82380  
 YAWR(5,5,2)=1.60182  
 THRUST(5,5,2)=12786.47266  
 ELEVAT(5,5,2)=-0.08002  
 DTAIL(5,5,2)=-0.00039  
 RUDDER(5,5,2)=-0.00265  
 THROTT(5,5,2)=26.63848  
 AILERO(5,5,2)=-0.00131  
 C C  
 MACH=1.2  
 LIFT(5,6,2)=81165.62500  
 DRAG(5,6,2)=28873.98047  
 BETA(5,6,2)=0.01047  
 ALPHA(5,6,2)=3.54796  
 PHI(5,6,2)=60.61400  
 THETA(5,6,2)=1.75178  
 ROLL(5,6,2)=-0.08342  
 PITCH(5,6,2)=2.37662  
 YAWR(5,6,2)=1.33839  
 THRUST(5,6,2)=22917.91602  
 ELEVAT(5,6,2)=-0.09375  
 DTAIL(5,6,2)=-0.00038  
 RUDDER(5,6,2)=-0.00201  
 THROTT(5,6,2)=47.74566  
 AILERO(5,6,2)=-0.00125  
 C C  
 MACH=1.6  
 LIFT(5,7,2)=81163.09375  
 DRAG(5,7,2)=36848.21484  
 BETA(5,7,2)=0.00461  
 ALPHA(5,7,2)=2.21020  
 PHI(5,7,2)=60.58639  
 THETA(5,7,2)=1.08988  
 ROLL(5,7,2)=-0.03893  
 PITCH(5,7,2)=1.78260  
 YAWR(5,7,2)=1.00500

THRUST(5,7,2)=36876.02734  
 ELEVAT(5,7,2)=-0.00904  
 DTAIL(5,7,2)=-0.00027  
 RUDDER(5,7,2)=-0.00167  
 THROTT(5,7,2)=76.82506  
 AILERO(5,7,2)=-0.00089  
 C C  
 MACH=1.8  
 LIFT(5,8,2)=81167.07031  
 DRAG(5,8,2)=41267.72656  
 BETA(5,8,2)=0.00370  
 ALPHA(5,8,2)=1.68706  
 PHI(5,8,2)=60.49760  
 THETA(5,8,2)=0.83422  
 ROLL(5,8,2)=-0.02640  
 PITCH(5,8,2)=1.57804  
 YAWR(5,8,2)=0.89290  
 THRUST(5,8,2)=41285.56250  
 ELEVAT(5,8,2)=0.00823  
 DTAIL(5,8,2)=-0.00019  
 RUDDER(5,8,2)=-0.00131  
 THROTT(5,8,2)=86.01159  
 AILERO(5,8,2)=-0.00062  
 C C  
 ALTITUDE=3500  
 MACH=0.6  
 LIFT(6,3,2)=81125.78906  
 DRAG(6,3,2)=25121.12891  
 BETA(6,3,2)=0.13794  
 ALPHA(6,3,2)=19.16866  
 PHI(6,3,2)=64.49668  
 THETA(6,3,2)=8.64137  
 ROLL(6,3,2)=-0.93476  
 PITCH(6,3,2)=5.55142  
 YAWR(6,3,2)=2.64829  
 THRUST(6,3,2)=26595.68555  
 ELEVAT(6,3,2)=-0.22453  
 DTAIL(6,3,2)=-0.00405  
 RUDDER(6,3,2)=-0.01256  
 THROTT(6,3,2)=55.40768  
 AILERO(6,3,2)=-0.01349  
 C C  
 MACH=0.8  
 LIFT(6,4,2)=81133.85938  
 DRAG(6,4,2)=9618.19336  
 BETA(6,4,2)=0.05329  
 ALPHA(6,4,2)=8.43525  
 PHI(6,4,2)=60.84182  
 THETA(6,4,2)=4.17941  
 ROLL(6,4,2)=-0.30493  
 PITCH(6,4,2)=3.64410  
 YAWR(6,4,2)=2.03313  
 THRUST(6,4,2)=9723.81641  
 ELEVAT(6,4,2)=-0.06713  
 DTAIL(6,4,2)=-0.00071  
 RUDDER(6,4,2)=-0.00512  
 THROTT(6,4,2)=20.25795  
 AILERO(6,4,2)=-0.00236  
 C C  
 MACH=1.0  
 LIFT(6,5,2)=81127.73438  
 DRAG(6,5,2)=12013.45215  
 BETA(6,5,2)=0.02288

ALPHA (6, 5, 2) = 5.21196  
 PHI (6, 5, 2) = 60.54324  
 THETA (6, 5, 2) = 2.58840  
 ROLL (6, 5, 2) = -0.15036  
 PITCH (6, 5, 2) = 2.89616  
 YAW (6, 5, 2) = 1.63569  
 THRUST (6, 5, 2) = 12063.65234  
 ELEVAT (6, 5, 2) = -0.10903  
 DTAIL (6, 5, 2) = -0.00048  
 RUDDER (6, 5, 2) = -0.00287  
 THROTT (6, 5, 2) = 25.13261  
 AILERO (6, 5, 2) = -0.00161  
  
 MACH=1.2  
 LIFT (6, 6, 2) = 81127.46875  
 DRAG (6, 6, 2) = 20889.64648  
 BETA (6, 6, 2) = 0.01179  
 ALPHA (6, 6, 2) = 0.01179  
 PHI (6, 6, 2) = 4.55320  
 THETA (6, 6, 2) = 2.23933  
 ROLL (6, 6, 2) = -0.10940  
 PITCH (6, 6, 2) = 2.44082  
 YAW (6, 6, 2) = 1.36746  
 THRUST (6, 6, 2) = 20955.77930  
 ELEVAT (6, 6, 2) = -0.13913  
 DTAIL (6, 6, 2) = -0.00049  
 RUDDER (6, 6, 2) = -0.00218  
 THROTT (6, 6, 2) = 43.65788  
 AILERO (6, 6, 2) = -0.00162  
  
 MACH=1.6  
 LIFT (6, 8, 2) = 81130.55469  
 DRAG (6, 8, 2) = 29889.12109  
 BETA (6, 8, 2) = 0.00470  
 ALPHA (6, 8, 2) = 2.85663  
 PHI (6, 8, 2) = 60.62601  
 THETA (6, 8, 2) = 1.40618  
 ROLL (6, 8, 2) = -0.05139  
 PITCH (6, 8, 2) = 1.82440  
 YAW (6, 8, 2) = 1.02891  
 THRUST (6, 8, 2) = 29926.24805  
 ELEVAT (6, 8, 2) = -0.02997  
 DTAIL (6, 8, 2) = -0.00032  
 RUDDER (6, 8, 2) = -0.00177  
 THROTT (6, 8, 2) = 62.34635  
 AILERO (6, 8, 2) = -0.00107  
  
 MACH=1.8  
 LIFT (6, 9, 2) = 81120.92969  
 DRAG (6, 9, 2) = 36804.26953  
 BETA (6, 9, 2) = 0.00380  
 ALPHA (6, 9, 2) = 2.19715  
 PHI (6, 9, 2) = 60.58247  
 THETA (6, 9, 2) = 1.08289  
 ROLL (6, 9, 2) = -0.03514  
 PITCH (6, 9, 2) = 1.61914  
 YAW (6, 9, 2) = 0.91299  
 THRUST (6, 9, 2) = 36831.98828  
 ELEVAT (6, 9, 2) = -0.00821  
 DTAIL (6, 9, 2) = -0.00222  
 RUDDER (6, 9, 2) = -0.00138  
 THROTT (6, 9, 2) = 76.73331  
 AILERO (6, 9, 2) = -0.00074

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ALTITUDE=40000

MACH=0.8  
 LIFT (7, 4, 2) = 81068.21875  
 DRAG (7, 4, 2) = 14734.15625  
 BETA (7, 4, 2) = 0.06168  
 ALPHA (7, 4, 2) = 11.35962  
 PHI (7, 4, 2) = 61.64415  
 THETA (7, 4, 2) = 5.50539  
 ROLL (7, 4, 2) = -0.41309  
 PITCH (7, 4, 2) = 3.77167  
 YAW (7, 4, 2) = 2.03558  
 THRUST (7, 4, 2) = 15028.41602  
 ELEVAT (7, 4, 2) = -0.09250  
 DTAIL (7, 4, 2) = 0.00128  
 RUDDER (7, 4, 2) = -0.00586  
 THROTT (7, 4, 2) = 31.30920  
 AILERO (7, 4, 2) = -0.00425  
  
 MACH=1.0  
 LIFT (7, 5, 2) = 81090.48438  
 DRAG (7, 5, 2) = 13232.69336  
 BETA (7, 5, 2) = 0.02454  
 ALPHA (7, 5, 2) = 6.61524  
 PHI (7, 5, 2) = 60.77803  
 THETA (7, 5, 2) = 3.26198  
 ROLL (7, 5, 2) = -0.19166  
 PITCH (7, 5, 2) = 2.93486  
 YAW (7, 5, 2) = 1.64171  
 THRUST (7, 5, 2) = 13321.38086  
 ELEVAT (7, 5, 2) = -0.14653  
 DTAIL (7, 5, 2) = -0.00058  
 RUDDER (7, 5, 2) = -0.00304  
 THROTT (7, 5, 2) = 27.75288  
 AILERO (7, 5, 2) = -0.00195  
  
 MACH=1.2  
 LIFT (7, 6, 2) = 81088.27344  
 DRAG (7, 6, 2) = 19292.47070  
 BETA (7, 6, 2) = 0.01306  
 ALPHA (7, 6, 2) = 5.85336  
 PHI (7, 6, 2) = 60.91299  
 THETA (7, 6, 2) = 2.86456  
 ROLL (7, 6, 2) = -0.14127  
 PITCH (7, 6, 2) = 2.46714  
 YAW (7, 6, 2) = 1.37246  
 THRUST (7, 6, 2) = 19393.58789  
 ELEVAT (7, 6, 2) = -0.19502  
 DTAIL (7, 6, 2) = -0.00064  
 RUDDER (7, 6, 2) = -0.00230  
 THROTT (7, 6, 2) = 40.40331  
 AILERO (7, 6, 2) = -0.00214  
  
 MACH=1.4  
 LIFT (7, 7, 2) = 81088.53906  
 DRAG (7, 7, 2) = 21800.05273  
 BETA (7, 7, 2) = 0.00781  
 ALPHA (7, 7, 2) = 4.52625  
 PHI (7, 7, 2) = 60.76378  
 THETA (7, 7, 2) = 2.22100  
 ROLL (7, 7, 2) = -0.09352  
 PITCH (7, 7, 2) = 2.10425  
 YAW (7, 7, 2) = 1.17777

C C

C C

THRUST (7, 7, 2) = 21868.25195  
ELEVAT (7, 7, 2) = -0.10316  
DTAIL (7, 7, 2) = -0.00046  
RUDDER (7, 7, 2) = -0.00198  
THROTT (7, 7, 2) = 45.55886  
AILERO (7, 7, 2) = -0.00154

MACH=1.6

LIFT (7, 8, 2) = 81087.07031  
DRAG (7, 8, 2) = 24260.49805  
BETA (7, 8, 2) = 0.00457  
ALPHA (7, 8, 2) = 3.69707  
PHI (7, 8, 2) = 60.67663  
THETA (7, 8, 2) = 1.81650  
ROLLR (7, 8, 2) = -0.06677  
PITCHR (7, 8, 2) = 1.83548  
YAWR (7, 8, 2) = 1.03101  
THRUST (7, 8, 2) = 24311.77148  
ELEVAT (7, 8, 2) = -0.05722  
DTAIL (7, 8, 2) = -0.00038  
RUDDER (7, 8, 2) = -0.00181  
THROTT (7, 8, 2) = 50.64952  
AILERO (7, 8, 2) = -0.00127

MACH=1.8

LIFT (7, 9, 2) = 81097.53125  
DRAG (7, 9, 2) = 25682.07422  
BETA (7, 9, 2) = 0.00376  
ALPHA (7, 9, 2) = 2.86130  
PHI (7, 9, 2) = 60.62327  
THETA (7, 9, 2) = 1.40777  
ROLLR (7, 9, 2) = -0.04594  
PITCHR (7, 9, 2) = 1.62883  
YAWR (7, 9, 2) = 0.91693  
THRUST (7, 9, 2) = 29718.66406  
ELEVAT (7, 9, 2) = -0.02962  
DTAIL (7, 9, 2) = -0.00026  
RUDDER (7, 9, 2) = -0.00141  
THROTT (7, 9, 2) = 61.91388  
AILERO (7, 9, 2) = -0.00086

MACH=2.0

LIFT (7, 10, 2) = 81064.25000  
DRAG (7, 10, 2) = 35739.31250  
BETA (7, 10, 2) = 0.00311  
ALPHA (7, 10, 2) = 2.26260  
PHI (7, 10, 2) = 60.58380  
THETA (7, 10, 2) = 1.11442  
ROLLR (7, 10, 2) = -0.03269  
PITCHR (7, 10, 2) = 1.46387  
YAWR (7, 10, 2) = 0.82539  
THRUST (7, 10, 2) = 35767.04688  
ELEVAT (7, 10, 2) = -0.00996  
DTAIL (7, 10, 2) = -0.00018  
RUDDER (7, 10, 2) = -0.00113  
THROTT (7, 10, 2) = 74.51468  
AILERO (7, 10, 2) = -0.00061

ALTITUDE=45000

MACH=0.8

LIFT (8, 4, 2) = 81050.51563  
DRAG (8, 4, 2) = 21814.13477  
BETA (8, 4, 2) = 0.08002

ALPHA (8, 4, 2) = 16.28868  
PHI (8, 4, 2) = 63.35456  
THETA (8, 4, 2) = 7.53903  
ROLLR (8, 4, 2) = -0.59440  
PITCHR (8, 4, 2) = 4.01430  
YAWR (8, 4, 2) = 2.01419  
THRUST (8, 4, 2) = 22726.44922  
ELEVAT (8, 4, 2) = -0.19221  
DTAIL (8, 4, 2) = -0.00216  
RUDDER (8, 4, 2) = -0.00829  
THROTT (8, 4, 2) = 47.34677  
AILERO (8, 4, 2) = -0.00718

MACH=1.0

LIFT (8, 5, 2) = 81051.78125  
DRAG (8, 5, 2) = 14852.18457  
BETA (8, 5, 2) = 0.02836  
ALPHA (8, 5, 2) = 8.56593  
PHI (8, 5, 2) = 61.16088  
THETA (8, 5, 2) = 4.18251  
ROLLR (8, 5, 2) = -0.24830  
PITCHR (8, 5, 2) = 2.97428  
YAWR (8, 5, 2) = 1.63777  
THRUST (8, 5, 2) = 15019.90723  
ELEVAT (8, 5, 2) = -0.19276  
DTAIL (8, 5, 2) = -0.00070  
RUDDER (8, 5, 2) = -0.00349  
THROTT (8, 5, 2) = 31.29147  
AILERO (8, 5, 2) = -0.00233

MACH=1.2

LIFT (8, 6, 2) = 81040.35938  
DRAG (8, 6, 2) = 18841.87109  
BETA (8, 6, 2) = 0.01485  
ALPHA (8, 6, 2) = 7.43313  
PHI (8, 6, 2) = 61.17528  
THETA (8, 6, 2) = 3.61231  
ROLLR (8, 6, 2) = -0.17937  
PITCHR (8, 6, 2) = 2.48918  
YAWR (8, 6, 2) = 1.56984  
THRUST (8, 6, 2) = 19001.48438  
ELEVAT (8, 6, 2) = -0.26022  
DTAIL (8, 6, 2) = -0.00085  
RUDDER (8, 6, 2) = -0.00247  
THROTT (8, 6, 2) = 39.58643  
AILERO (8, 6, 2) = -0.00284

MACH=1.4

LIFT (8, 7, 2) = 81051.44531  
DRAG (8, 7, 2) = 20431.61133  
BETA (8, 7, 2) = 0.00858  
ALPHA (8, 7, 2) = 5.86934  
PHI (8, 7, 2) = 60.96077  
THETA (8, 7, 2) = 2.86419  
ROLLR (8, 7, 2) = -0.12125  
PITCHR (8, 7, 2) = 2.11879  
YAWR (8, 7, 2) = 1.17636  
THRUST (8, 7, 2) = 20539.75781  
ELEVAT (8, 7, 2) = -0.16447  
DTAIL (8, 7, 2) = -0.00057  
RUDDER (8, 7, 2) = -0.00211  
THROTT (8, 7, 2) = 42.79116  
AILERO (8, 7, 2) = -0.00191

C MACH=1.6  
 LIFT(8,8,2)=81062.45313  
 DRAG(8,8,2)=22251.68945  
 BETA(8,8,2)=0.00487  
 ALPHA(8,8,2)=4.82793  
 PHI(8,8,2)=60.83395  
 THETA(8,8,2)=2.36137  
 ROLL(8,8,2)=0.08717  
 PITCH(8,8,2)=1.84594  
 YAWR(8,8,2)=1.03023  
 THRUST(8,8,2)=22330.33594  
 ELEVAT(8,8,2)=0.09768  
 DTAIL(8,8,2)=0.00045  
 RUDDER(8,8,2)=0.00199  
 THROTT(8,8,2)=46.52153  
 AILERO(8,8,2)=0.00149  
  
 C MACH=1.8  
 LIFT(8,9,2)=81050.97656  
 DRAG(8,9,2)=24089.63086  
 BETA(8,9,2)=0.00361  
 ALPHA(8,9,2)=3.70487  
 PHI(8,9,2)=60.67407  
 THETA(8,9,2)=1.81964  
 ROLL(8,9,2)=0.05942  
 PITCH(8,9,2)=1.63054  
 YAWR(8,9,2)=0.91599  
 THRUST(8,9,2)=24140.02930  
 ELEVAT(8,9,2)=0.05695  
 DTAIL(8,9,2)=0.00030  
 RUDDER(8,9,2)=0.00143  
 THROTT(8,9,2)=50.29173  
 AILERO(8,9,2)=0.00101  
  
 C MACH=2.0  
 LIFT(8,10,2)=81048.16406  
 DRAG(8,10,2)=28837.06055  
 BETA(8,10,2)=0.00303  
 ALPHA(8,10,2)=2.94627  
 PHI(8,10,2)=60.62562  
 THETA(8,10,2)=1.44880  
 ROLL(8,10,2)=0.04253  
 PITCH(8,10,2)=1.46529  
 YAWR(8,10,2)=0.82478  
 THRUST(8,10,2)=28874.82813  
 ELEVAT(8,10,2)=0.03199  
 DTAIL(8,10,2)=0.00021  
 RUDDER(8,10,2)=0.00114  
 THROTT(8,10,2)=60.15589  
 AILERO(8,10,2)=0.00071  
  
 C ALTITUDE=50000  
  
 C MACH=1.0  
 LIFT(9,5,2)=81011.07813  
 DRAG(9,5,2)=18314.33203  
 BETA(9,5,2)=0.03350  
 ALPHA(9,5,2)=11.89992  
 PHI(9,5,2)=62.01752  
 THETA(9,5,2)=5.67674  
 ROLL(9,5,2)=0.34516  
 PITCH(9,5,2)=3.06640  
 YAWR(9,5,2)=1.62923  
 THRUST(9,5,2)=18716.57227

C DTAIL(9,5,2)=0.00097  
 RUDDER(9,5,2)=0.00406  
 THROTT(9,5,2)=38.99286  
 AILERO(9,5,2)=0.00324  
  
 C MACH=1.2  
 LIFT(9,6,2)=81011.61719  
 DRAG(9,6,2)=20272.80664  
 BETA(9,6,2)=0.01747  
 ALPHA(9,6,2)=9.28838  
 PHI(9,6,2)=61.61195  
 THETA(9,6,2)=4.46170  
 ROLL(9,6,2)=0.22433  
 PITCH(9,6,2)=2.52927  
 YAWR(9,6,2)=1.36689  
 THRUST(9,6,2)=20542.12695  
 ELEVAT(9,6,2)=0.03249  
 DTAIL(9,6,2)=0.00101  
 RUDDER(9,6,2)=0.00284  
 THROTT(9,6,2)=42.79610  
 AILERO(9,6,2)=0.00338  
  
 C MACH=1.4  
 LIFT(9,7,2)=80998.95313  
 DRAG(9,7,2)=19329.42383  
 BETA(9,7,2)=0.00972  
 ALPHA(9,7,2)=7.52520  
 PHI(9,7,2)=61.21634  
 THETA(9,7,2)=3.64804  
 ROLL(9,7,2)=0.15542  
 PITCH(9,7,2)=2.13654  
 YAWR(9,7,2)=1.17378  
 THRUST(9,7,2)=19496.93945  
 ELEVAT(9,7,2)=0.02307  
 DTAIL(9,7,2)=0.00072  
 RUDDER(9,7,2)=0.00230  
 THROTT(9,7,2)=40.61862  
 AILERO(9,7,2)=0.00239  
  
 C MACH=1.6  
 LIFT(9,8,2)=81013.95313  
 DRAG(9,8,2)=21250.80469  
 BETA(9,8,2)=0.00514  
 ALPHA(9,8,2)=6.29808  
 PHI(9,8,2)=61.07397  
 THETA(9,8,2)=3.06030  
 ROLL(9,8,2)=0.11373  
 PITCH(9,8,2)=1.86196  
 YAWR(9,8,2)=1.02896  
 THRUST(9,8,2)=21379.83008  
 ELEVAT(9,8,2)=0.015187  
 DTAIL(9,8,2)=0.00052  
 RUDDER(9,8,2)=0.00213  
 THROTT(9,8,2)=44.54131  
 AILERO(9,8,2)=0.00175  
  
 C MACH=1.8  
 LIFT(9,9,2)=81025.51563  
 DRAG(9,9,2)=22141.71875  
 BETA(9,9,2)=0.00384  
 ALPHA(9,9,2)=4.84486  
 PHI(9,9,2)=60.83381  
 THETA(9,9,2)=2.36878  
 ROLL(9,9,2)=0.07769

PITCH(9,9,2)=1.64000  
 YAWR(9,9,2)=0.91530  
 THRUST(9,9,2)=22221.16406  
 ELEVAT(9,9,2)=-0.09777  
 DTAIL(9,9,2)=-0.00035  
 RUDDER(9,9,2)=-0.00157  
 THROTT(9,9,2)=-46.29409  
 AILERO(9,9,2)=-0.00118

MACH=2.0  
 LIFT(9,10,2)=81011.79688  
 DRAG(9,10,2)=23407.07813  
 BETA(9,10,2)=0.00291  
 ALPHA(9,10,2)=3.81792  
 PHI(9,10,2)=60.67836  
 THETA(9,10,2)=1.87433  
 ROLL(9,10,2)=-0.05506  
 PITCHR(9,10,2)=1.46687  
 YAWR(9,10,2)=0.82390  
 THRUST(9,10,2)=23459.06055  
 ELEVAT(9,10,2)=-0.06024  
 DTAIL(9,10,2)=-0.00025  
 RUDDER(9,10,2)=-0.00116  
 THROTT(9,10,2)=48.87304  
 AILERO(9,10,2)=-0.00083

LOAD FACTOR=4.0

ALTITUDE=10000

MACH=0.6  
 LIFT(1,3,3)=162637.68750  
 DRAG(1,3,3)=28627.04688  
 BETA(1,3,3)=0.10448  
 ALPHA(1,3,3)=11.76942  
 PHI(1,3,3)=76.34757  
 THETA(1,3,3)=2.91887  
 ROLL(1,3,3)=-0.58383  
 PITCHR(1,3,3)=11.12673  
 YAWR(1,3,3)=2.70362  
 THRUST(1,3,3)=29241.77734  
 ELEVAT(1,3,3)=-0.13818  
 DTAIL(1,3,3)=-0.00264  
 RUDDER(1,3,3)=-0.00857  
 THROTT(1,3,3)=60.92037  
 AILERO(1,3,3)=-0.00882

MACH=0.8  
 LIFT(1,4,3)=162646.50000  
 DRAG(1,4,3)=15118.63184  
 BETA(1,4,3)=0.04100  
 ALPHA(1,4,3)=5.84141  
 PHI(1,4,3)=75.73713  
 THETA(1,4,3)=1.48378  
 ROLL(1,4,3)=-0.21646  
 PITCHR(1,4,3)=8.09903  
 YAWR(1,4,3)=2.05883  
 THRUST(1,4,3)=15197.54199  
 ELEVAT(1,4,3)=-0.04840  
 DTAIL(1,4,3)=-0.00053  
 RUDDER(1,4,3)=-0.00400  
 THROTT(1,4,3)=31.66155

AILERO(1,4,3)=-0.00178

MACH=1  
 LIFT(1,5,3)=162640.18750  
 DRAG(1,5,3)=27455.81641  
 BETA(1,5,3)=0.01957  
 ALPHA(1,5,3)=3.65306  
 PHI(1,5,3)=75.70924  
 THETA(1,5,3)=0.92187  
 ROLL(1,5,3)=-0.10774  
 PITCHR(1,5,3)=6.48842  
 YAWR(1,5,3)=1.65276  
 THRUST(1,5,3)=27511.68164  
 ELEVAT(1,5,3)=-0.06981  
 DTAIL(1,5,3)=-0.00035  
 RUDDER(1,5,3)=-0.00249  
 THROTT(1,5,3)=57.31601  
 AILERO(1,5,3)=-0.00117

MACH=1.2  
 LIFT(1,6,3)=162646.06250  
 DRAG(1,6,3)=47224.27344  
 BETA(1,6,3)=0.00972  
 ALPHA(1,6,3)=3.12740  
 PHI(1,6,3)=75.77425  
 THETA(1,6,3)=0.77869  
 ROLL(1,6,3)=-0.07625  
 PITCHR(1,6,3)=5.43777  
 YAWR(1,6,3)=1.37857  
 THRUST(1,6,3)=47294.63672  
 ELEVAT(1,6,3)=-0.07614  
 DTAIL(1,6,3)=-0.00032  
 RUDDER(1,6,3)=-0.00190  
 THROTT(1,6,3)=98.53049  
 AILERO(1,6,3)=-0.00107

ALTITUDE=15000

MACH=0.6  
 LIFT(2,3,3)=162566.85938  
 DRAG(2,3,3)=37908.89063  
 BETA(2,3,3)=0.12299  
 ALPHA(2,3,3)=15.11119  
 PHI(2,3,3)=76.88128  
 THETA(2,3,3)=3.63078  
 ROLL(2,3,3)=-0.75936  
 PITCHR(2,3,3)=11.65481  
 YAWR(2,3,3)=2.71617  
 THRUST(2,3,3)=39266.66406  
 ELEVAT(2,3,3)=-0.17338  
 DTAIL(2,3,3)=-0.00389  
 RUDDER(2,3,3)=-0.00962  
 THROTT(2,3,3)=81.80555  
 AILERO(2,3,3)=-0.01296

MACH=0.8  
 LIFT(2,4,3)=162567.48438  
 DRAG(2,4,3)=17767.24023  
 BETA(2,4,3)=0.04479  
 ALPHA(2,4,3)=7.07972  
 PHI(2,4,3)=75.83018  
 THETA(2,4,3)=1.78516  
 ROLL(2,4,3)=-0.26636  
 PITCHR(2,4,3)=8.28621

YAWR(2,4,3)=2.09209  
 THRUST(2,4,3)=17903.72070  
 ELEVAT(2,4,3)=-0.05968  
 DTAIL(2,4,3)=-0.00060  
 RUDDER(2,4,3)=-0.00432  
 THROTT(2,4,3)=37.29942  
 AILERO(2,4,3)=-0.00201

C C

MACH=1.0  
 LIFT(2,5,3)=162583.42188  
 DRAG(2,5,3)=25803.77148  
 BETA(2,5,3)=0.02083  
 ALPHA(2,5,3)=4.42098  
 PHI(2,5,3)=75.74406  
 THETA(2,5,3)=1.11096  
 ROLL(2,5,3)=-0.13244  
 PITCH(2,5,3)=6.61934  
 YAWR(2,5,3)=1.68183  
 THRUST(2,5,3)=25879.77734  
 ELEVAT(2,5,3)=-0.08744  
 DTAIL(2,5,3)=-0.00040  
 RUDDER(2,5,3)=-0.00264  
 THROTT(2,5,3)=53.91620  
 AILERO(2,5,3)=-0.00134

C C

MACH=1.2  
 LIFT(2,6,3)=162565.84375  
 DRAG(2,6,3)=42154.65625  
 BETA(2,6,3)=0.01045  
 ALPHA(2,6,3)=3.80848  
 PHI(2,6,3)=75.80418  
 THETA(2,6,3)=0.94543  
 ROLL(2,6,3)=-0.09442  
 PITCH(2,6,3)=5.54684  
 YAWR(2,6,3)=1.40314  
 THRUST(2,6,3)=42247.54297  
 ELEVAT(2,6,3)=-0.10646  
 DTAIL(2,6,3)=-0.00039  
 RUDDER(2,6,3)=-0.00198  
 THROTT(2,6,3)=88.01571  
 AILERO(2,6,3)=-0.00129

C C C C

ALTITUDE=20000

MACH=0.8  
 LIFT(3,4,3)=162532.15625  
 DRAG(3,4,3)=21283.40039  
 BETA(3,4,3)=0.05311  
 ALPHA(3,4,3)=8.88117  
 PHI(3,4,3)=75.99001  
 THETA(3,4,3)=2.21849  
 ROLL(3,4,3)=-0.33979  
 PITCH(3,4,3)=8.51030  
 YAWR(3,4,3)=2.12343  
 THRUST(3,4,3)=21542.14063  
 ELEVAT(3,4,3)=-0.07606  
 DTAIL(3,4,3)=-0.00076  
 RUDDER(3,4,3)=-0.00509  
 THROTT(3,4,3)=44.87946  
 AILERO(3,4,3)=-0.00254

C C

MACH=1.0  
 LIFT(3,5,3)=162492.79688  
 DRAG(3,5,3)=24331.69727

BETA(3,5,3)=0.02259  
 ALPHA(3,5,3)=5.41493  
 PHI(3,5,3)=75.79254  
 THETA(3,5,3)=1.35472  
 ROLL(3,5,3)=-0.16493  
 PITCH(3,5,3)=6.76102  
 YAWR(3,5,3)=1.71174  
 THRUST(3,5,3)=24440.76367  
 ELEVAT(3,5,3)=-0.11539  
 DTAIL(3,5,3)=-0.00048  
 RUDDER(3,5,3)=-0.00283  
 THROTT(3,5,3)=50.91826  
 AILERO(3,5,3)=-0.00161

C C C C

ALTITUDE=25000

MACH=0.8  
 LIFT(4,4,3)=162410.68750  
 DRAG(4,4,3)=30147.55078  
 BETA(4,4,3)=0.06214  
 ALPHA(4,4,3)=11.54214  
 PHI(4,4,3)=76.34508  
 THETA(4,4,3)=2.82161  
 ROLL(4,4,3)=-0.44875  
 PITCH(4,4,3)=8.84764  
 YAWR(4,4,3)=2.14945  
 THRUST(4,4,3)=30769.78320  
 ELEVAT(4,4,3)=-0.10122  
 DTAIL(4,4,3)=-0.00132  
 RUDDER(4,4,3)=-0.00590  
 THROTT(4,4,3)=64.10371  
 AILERO(4,4,3)=-0.00440

C C

MACH=1.0  
 LIFT(4,5,3)=162413.93750  
 DRAG(4,5,3)=27084.75781  
 BETA(4,5,3)=0.02477  
 ALPHA(4,5,3)=6.68738  
 PHI(4,5,3)=75.90145  
 THETA(4,5,3)=1.66015  
 ROLL(4,5,3)=-0.20731  
 PITCH(4,5,3)=6.93739  
 YAWR(4,5,3)=1.74237  
 THRUST(4,5,3)=27270.30469  
 ELEVAT(4,5,3)=-0.14957  
 DTAIL(4,5,3)=-0.00059  
 RUDDER(4,5,3)=-0.00306  
 THROTT(4,5,3)=56.81313  
 AILERO(4,5,3)=-0.00196

C C

MACH=1.2  
 LIFT(4,6,3)=162394.62500  
 DRAG(4,6,3)=39205.00391  
 BETA(4,6,3)=0.01320  
 ALPHA(4,6,3)=5.92501  
 PHI(4,6,3)=75.95746  
 THETA(4,6,3)=1.45537  
 ROLL(4,6,3)=-0.15232  
 PITCH(4,6,3)=5.81632  
 YAWR(4,6,3)=1.45476  
 THRUST(4,6,3)=39415.80859  
 ELEVAT(4,6,3)=-0.19933  
 DTAIL(4,6,3)=-0.00065  
 RUDDER(4,6,3)=-0.00233



THROTT(4,6,3)=82.11627  
AILERO(4,6,3)=-0.00217

C C

MACH=1.4  
LIFT(4,7,3)=162410.04688  
DRAG(4,7,3)=41133.01172  
BETA(4,7,3)=0.00789  
ALPHA(4,7,3)=4.58291  
PHI(4,7,3)=75.86076  
THETA(4,7,3)=1.12943  
ROLLR(4,7,3)=-0.10083  
PITCHR(4,7,3)=4.95938  
YAWR(4,7,3)=1.24932  
THRUST(4,7,3)=41264.51563  
ELEVAT(4,7,3)=-0.10593  
DTAIL(4,7,3)=-0.00047  
RUDDER(4,7,3)=-0.00201  
THROTT(4,7,3)=85.96774  
AILERO(4,7,3)=-0.00156

ALITUDE=30000

C C C C

MACH=1.0  
LIFT(5,5,3)=162333.00000  
DRAG(5,5,3)=29875.87305  
BETA(5,5,3)=0.02956  
ALPHA(5,5,3)=8.49303  
PHI(5,5,3)=76.07033  
THETA(5,5,3)=2.08773  
ROLLR(5,5,3)=-0.26833  
PITCHR(5,5,3)=7.14424  
YAWR(5,5,3)=1.77195  
THRUST(5,5,3)=30206.55469  
ELEVAT(5,5,3)=-0.19283  
DTAIL(5,5,3)=-0.00072  
RUDDER(5,5,3)=-0.00364  
THROTT(5,5,3)=62.93032  
AILERO(5,5,3)=-0.00241

C C

MACH=1.2  
LIFT(5,6,3)=162331.64063  
DRAG(5,6,3)=38183.48438  
BETA(5,6,3)=0.01547  
ALPHA(5,6,3)=7.39328  
PHI(5,6,3)=76.07425  
THETA(5,6,3)=1.80379  
ROLLR(5,6,3)=-0.19381  
PITCHR(5,6,3)=5.97335  
YAWR(5,6,3)=1.48110  
THRUST(5,6,3)=38503.61328  
ELEVAT(5,6,3)=-0.26033  
DTAIL(5,6,3)=-0.00088  
RUDDER(5,6,3)=-0.00258  
THROTT(5,6,3)=80.21586  
AILERO(5,6,3)=-0.00294

ALITUDE=35000

C C C C

MACH=1.0  
LIFT(6,5,3)=162252.50000  
DRAG(6,5,3)=36575.35938  
BETA(6,5,3)=0.03595  
ALPHA(6,5,3)=11.64956  
PHI(6,5,3)=76.45950

THETA(6,5,3)=2.79922  
ROLLR(6,5,3)=-0.37485  
PITCHR(6,5,3)=7.45342  
YAWR(6,5,3)=1.79498  
THRUST(6,5,3)=37344.71484  
ELEVAT(6,5,3)=-0.26145  
DTAIL(6,5,3)=-0.00104  
RUDDER(6,5,3)=-0.00436  
THROTT(6,5,3)=77.80149  
AILERO(6,5,3)=-0.00345

C C

MACH=1.2  
LIFT(6,6,3)=162255.09375  
DRAG(6,6,3)=39361.97656  
BETA(6,6,3)=0.01884  
ALPHA(6,6,3)=9.15001  
PHI(6,6,3)=76.25133  
THETA(6,6,3)=2.21074  
ROLLR(6,6,3)=-0.24489  
PITCHR(6,6,3)=6.16192  
YAWR(6,6,3)=1.50766  
THRUST(6,6,3)=39868.64453  
ELEVAT(6,6,3)=-0.32308  
DTAIL(6,6,3)=-0.00109  
RUDDER(6,6,3)=-0.00306  
THROTT(6,6,3)=83.05968  
AILERO(6,6,3)=-0.00364

C C

MACH=1.4  
LIFT(6,7,3)=162256.62500  
DRAG(6,7,3)=39437.30469  
BETA(6,7,3)=0.01052  
ALPHA(6,7,3)=7.40734  
PHI(6,7,3)=76.08894  
THETA(6,7,3)=1.80054  
ROLLR(6,7,3)=-0.16967  
PITCHR(6,7,3)=5.23897  
YAWR(6,7,3)=1.29759  
THRUST(6,7,3)=39768.84766  
ELEVAT(6,7,3)=-0.22685  
DTAIL(6,7,3)=-0.00077  
RUDDER(6,7,3)=-0.00250  
THROTT(6,7,3)=82.85177  
AILERO(6,7,3)=-0.00256

C C

MACH=1.6  
LIFT(6,8,3)=162255.81250  
DRAG(6,8,3)=40620.19922  
BETA(6,8,3)=0.00560  
ALPHA(6,8,3)=6.18902  
PHI(6,8,3)=75.99258  
THETA(6,8,3)=1.50901  
ROLLR(6,8,3)=-0.12386  
PITCHR(6,8,3)=4.56184  
YAWR(6,8,3)=1.13802  
THRUST(6,8,3)=40858.48438  
ELEVAT(6,8,3)=-0.14759  
DTAIL(6,8,3)=-0.00056  
RUDDER(6,8,3)=-0.00233  
THROTT(6,8,3)=85.12185  
AILERO(6,8,3)=-0.00188

C C

MACH=1.8  
LIFT(6,9,3)=161889.98438

DRAG(6,9,3)=41801.17969  
 BETA(6,9,3)=0.00421  
 ALPHA(6,9,3)=4.75071  
 PHI(6,9,3)=75.87917  
 THETA(6,9,3)=1.16562  
 ROLL(6,9,3)=0.08454  
 PITCH(6,9,3)=4.02938  
 YAWR(6,9,3)=1.01367  
 THRUST(6,9,3)=41908.77734  
 ELEVAT(6,9,3)=0.09337  
 DTAIL(6,9,3)=0.00038  
 RUDDER(6,9,3)=0.00172  
 THROTT(6,9,3)=87.30996  
 AILERO(6,9,3)=0.00127  
 -----  
 ALTITUDE=40000  
 -----  
 MACH=1.2  
 LIFT(7,6,3)=162178.01563  
 DRAG(7,6,3)=44539.42578  
 BETA(7,6,3)=0.02214  
 ALPHA(7,6,3)=11.54755  
 PHI(7,6,3)=76.57835  
 THETA(7,6,3)=2.73715  
 ROLL(7,6,3)=0.30984  
 PITCH(7,6,3)=6.30381  
 YAWR(7,6,3)=1.50430  
 THRUST(7,6,3)=45459.64453  
 ELEVAT(7,6,3)=0.38773  
 DTAIL(7,6,3)=0.00124  
 RUDDER(7,6,3)=0.00357  
 THROTT(7,6,3)=94.70759  
 AILERO(7,6,3)=0.00412  
 -----  
 MACH=1.4  
 LIFT(7,7,3)=162179.32813  
 DRAG(7,7,3)=42382.75391  
 BETA(7,7,3)=0.01254  
 ALPHA(7,7,3)=9.68663  
 PHI(7,7,3)=76.34529  
 THETA(7,7,3)=2.31986  
 ROLL(7,7,3)=0.22251  
 PITCH(7,7,3)=5.33738  
 YAWR(7,7,3)=1.29664  
 THRUST(7,7,3)=42995.42969  
 ELEVAT(7,7,3)=0.31858  
 DTAIL(7,7,3)=0.00092  
 RUDDER(7,7,3)=0.00301  
 THROTT(7,7,3)=89.57381  
 AILERO(7,7,3)=0.00306  
 -----  
 MACH=1.6  
 LIFT(7,8,3)=162179.10938  
 DRAG(7,8,3)=39487.53516  
 BETA(7,8,3)=0.00633  
 ALPHA(7,8,3)=8.01523  
 PHI(7,8,3)=76.14453  
 THETA(7,8,3)=1.93753  
 ROLL(7,8,3)=0.16092  
 PITCH(7,8,3)=4.61842  
 YAWR(7,8,3)=1.13914  
 THRUST(7,8,3)=39877.09766  
 ELEVAT(7,8,3)=0.21663  
 DTAIL(7,8,3)=0.00066

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RUDDER(7,8,3)=-0.00270  
 THROTT(7,8,3)=83.07729  
 AILERO(7,8,3)=-0.00222  
 -----  
 MACH=1.8  
 LIFT(7,9,3)=162177.53125  
 DRAG(7,9,3)=40454.16016  
 BETA(7,9,3)=0.00447  
 ALPHA(7,9,3)=6.19740  
 PHI(7,9,3)=75.99183  
 THETA(7,9,3)=1.51004  
 ROLL(7,9,3)=0.11066  
 PITCH(7,9,3)=4.07314  
 YAWR(7,9,3)=1.01616  
 THRUST(7,9,3)=40691.96875  
 ELEVAT(7,9,3)=0.14666  
 DTAIL(7,9,3)=0.00045  
 RUDDER(7,9,3)=0.00186  
 THROTT(7,9,3)=84.77493  
 AILERO(7,9,3)=0.00150  
 -----  
 MACH=2.0  
 LIFT(7,10,3)=162178.53125  
 DRAG(7,10,3)=41534.83984  
 BETA(7,10,3)=0.00346  
 ALPHA(7,10,3)=4.90456  
 PHI(7,10,3)=75.89043  
 THETA(7,10,3)=1.20174  
 ROLL(7,10,3)=0.07885  
 PITCH(7,10,3)=3.64528  
 YAWR(7,10,4)=0.91628  
 THRUST(7,10,4)=41687.55859  
 ELEVAT(7,10,4)=0.09810  
 DTAIL(7,10,4)=0.00032  
 RUDDER(7,10,4)=0.00142  
 THROTT(7,10,4)=86.84908  
 AILERO(7,10,4)=0.00106  
 -----  
 MACH=1.8  
 LIFT(8,9,3)=162214.70313  
 DRAG(8,9,3)=39377.40234  
 BETA(8,9,3)=0.00512  
 ALPHA(8,9,3)=8.03954  
 PHI(8,9,3)=76.14635  
 THETA(8,9,3)=1.94202  
 ROLL(8,9,3)=0.14332  
 PITCH(8,9,3)=4.10369  
 YAWR(8,9,3)=1.01204  
 THRUST(8,9,3)=39840.25391  
 ELEVAT(8,9,3)=0.21633  
 DTAIL(8,9,3)=0.00053  
 RUDDER(8,9,3)=0.00214  
 THROTT(8,9,3)=83.00053  
 AILERO(8,9,3)=0.00175  
 -----  
 MACH=2.0  
 LIFT(8,10,3)=162104.34375  
 DRAG(8,10,3)=40179.00000  
 BETA(8,10,3)=0.00364  
 ALPHA(8,10,3)=6.38152  
 PHI(8,10,3)=76.00535  
 THETA(8,10,3)=1.55284  
 ROLL(8,10,3)=0.10244

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PITCH(8,10,3)=-3.66663
YAWR(8,10,3)=0.91383
THRUST(8,10,3)=40429.54297
ELEVAT(8,10,3)=-0.15258
DTAIL(8,10,3)=-0.00037
RUDDER(8,10,3)=-0.00152
THROTT(8,10,3)=84.22822
AILERO(8,10,3)=-0.00124
ALITUDE=50000

MACH=1.8
LIFT(9,9,3)=162025.79688
DRAG(9,9,3)=45569.39453
BETA(9,9,3)=0.00601
ALPHA(9,9,3)=10.78515
PHI(9,9,3)=76.50813
THETA(9,9,3)=2.55066
ROLLR(9,9,3)=-0.19182
PITCHR(9,9,3)=4.18727
YAWR(9,9,3)=1.00465
THRUST(9,9,3)=46389.91016
ELEVAT(9,9,3)=-0.33956
DTAIL(9,9,3)=-0.00069
RUDDER(9,9,3)=-0.00255
THROTT(9,9,3)=96.64565
AILERO(9,9,3)=-0.00229

MACH=2.0
LIFT(9,10,3)=163487.95313
DRAG(9,10,3)=39456.94531
BETA(9,10,3)=0.00408
ALPHA(9,10,3)=8.40564
PHI(9,10,3)=76.18526
THETA(9,10,3)=2.02481
ROLLR(9,10,3)=-0.13469
PITCHR(9,10,3)=3.69943
YAWR(9,10,3)=0.90968
THRUST(9,10,3)=39840.79688
ELEVAT(9,10,3)=-0.232086
DTAIL(9,10,3)=-0.00043
RUDDER(9,10,3)=-0.00178
THROTT(9,10,3)=83.00166
AILERO(9,10,3)=-0.00145

CONTINUE

LINEAR INTERPOLATION

QUANTITIES TO BE INTERPOLATED
BETA, LIFT, THRUST, DRAG, PHI, THETA, P, Q, R, THROTTLE
ELEVATOR, AILERON, RUDDER AND DIFFERENTIAL TAIL

SEARCH FOR THE LOCATION OF THE GIVEN MACH-ALTITUDE
LOAD FACTOR POINT IN THE GIVEN 3-D TABLE

LOCATE ALTITUDE-MACH INDICES FIRST
I-ALTITUDE, J-MACH NO, K-LOAD FACTOR
I=LASTI
IF (H.GT.ALTI(I+1)) I=I+1
IF (I.GT.IMAX) GO TO 10000
IF (H.GT.ALTI(I+1)) GO TO 10

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C C C C

C C

C 100

C C

10

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20 IF (H.LT.ALTI(I)) I=I-1
IF (I.LT.1) GO TO 10000
IF (H.LT.ALTI(I)) GO TO 20
LASTI=I

C

30 J=LASTI
IF (XM.GT.XMACH(J+1)) J=J+1
IF (J.GT.JMAX) GO TO 10000
IF (XM.GT.XMACH(J+1)) GO TO 30
IF (XM.LT.XMACH(J)) J=J-1
IF (J.LT.1) GO TO 10000
IF (XM.LT.XMACH(J)) GO TO 40
LASTJ=J

C

40 IPO=I+1
JPO=J+1
IOP=I-1
JOP=J-1
IALP=1

C

C LOCATE LOAD FACTOR INDEX GIVEN EITHER THE
LOAD FACTOR OR THE ANGLE OF ATTACK

C IF XLFAC<0: TURNING FLIGHT WITH ALFA GIVEN
C IF XLFAC>1: TURNING FLIGHT WITH LOAD FACTOR GIVEN
C IF XLFAC=1: STRAIGHT AND LEVEL TRIMS
C
C IF (XLFAC.LT.0.0 OR XLFAC.GT.1.0) GO TO 50
K=1
SYMMETRIC FLIGHT, DO NOT INTERPOLATE ALONG THE
LOAD FACTOR DIRECTION
IOP=0
XLFAC=1.00
GO TO 60
C IF (XLFAC.GT.1.0) GO TO 70
ALFA IN TURNING FLIGHT IS GIVEN
IALP=0
C
C DETERMINE THE REFERENCE VALUES OF ANGLE OF ATTACK
AT THE GIVEN H,M FOR ALL THE REFERENCE LOAD FACTORS
AND STORE IN THE 1-D ARRAY ALTEMP.
DO 800 KT=1, KMAX
CALL LINT3D(H,XM,XN(KT),ALTI(I),ALT(JPO),XMACH(J),
1 XMACH(JPO),XN(KT),XN(KT),ALPHA(IPO,J,KT),ALPHA(I,JPO,KT),
2 ALPHA(I,J,KT),ALPHA(IPO,J,KT),ALPHA(I,J,KT),ALPHA(IPO,J,KT),
3 ALPHA(IPO,JPO,KT),ALPHA(I,J,KT),ALPHA(IPO,J,KT),
4 ALPHA(I,JPO,KT),ALPHA(IPO,JPO,KT),ALTEMP(KT),IOP)
K=LASTK
IF (ALP.GT.ALTEMP(K+1)) K=K+1
IF (K.GT.KMAX) GO TO 10000
IF (ALP.GT.ALTEMP(K+1)) GO TO 80
IF (ALP.LT.ALTEMP(K)) K=K-1
IF (K.LT.1) GO TO 10000
IF (ALP.LT.ALTEMP(K)) GO TO 90

C COMPUTE THE LOAD FACTOR CORRESPONDING TO THE
C GIVEN ANGLE OF ATTACK
C
C DXL=(XN(K+1)-XN(K))/(ALTEMP(K+1)-ALTEMP(K))
C XLFAC=XN(K)+DXL*(ALP-ALTEMP(K))
C LASTK=K
C GO TO 60
C LOAD FACTOR IN TURNING FLIGHT IS GIVEN
C K=LASTK
70

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140 IF (XLFAC.GT.XN(K+1))K=K+1
    IF (K.GT.10000)GO TO 10000
    IF (XLFAC.GT.XN(K+1))GO TO 140
110 IF (XLFAC.LT.XN(K))K=K-1
    IF (K.LT.1)GO TO 10000
    IF (XLFAC.LT.XN(K))GO TO 110
    LASTK=K
    CONTINUE
60 KPO=K+1
    XDAT(1)=ALT(1)
    XDAT(2)=ALT(IPO)
    YDAT(1)=XWACH(1)
    YDAT(2)=XWACH(IPO)
    ZDAT(1)=XN(K)
    ZDAT(2)=XN(IPO)
    XDAT(2)=XN(KPO)
    INTERPOLATE ANGLE OF ATTACK IF THE LOAD FACTOR IS
    GIVEN
    IF (IALP.EQ.0)GO TO 120
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 ALPHA(I,J,K),ALPHA(IPO,J,K),ALPHA(I,J,KPO),
    3 ALPHA(I,JPO,K),ALPHA(I,J,KPO),ALPHA(IPO,J,KPO),
    4 ALPHA(I,JPO,KPO),ALPHA(IPO,JPO,KPO),ALP,IOP,T)
    CONTINUE
120 CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 THRUST(I,J,K),THRUST(IPO,J,K),THRUST(I,J,KPO),
    3 THRUST(I,JPO,K),THRUST(I,J,KPO),THRUST(IPO,J,KPO),
    4 THRUST(I,JPO,KPO),THRUST(IPO,JPO,KPO),THRST,IOP,T)
    CONTINUE
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 DRAG(I,J,K),DRAG(IPO,J,K),DRAG(I,J,KPO),
    3 DRAG(I,JPO,K),DRAG(I,J,KPO),DRAG(IPO,J,KPO),
    4 DRAG(I,JPO,KPO),DRAG(IPO,JPO,KPO),DRG,IOP,T)
    CONTINUE
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 PHI(I,J,K),PHI(IPO,J,K),PHI(I,J,KPO),
    3 PHI(I,JPO,K),PHI(I,J,KPO),PHI(IPO,J,KPO),
    4 PHI(I,JPO,KPO),PHI(IPO,JPO,KPO),PHIO,IOP,T)
    CONTINUE
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 THETA(I,J,K),THETA(IPO,J,K),THETA(I,J,KPO),
    3 THETA(I,JPO,K),THETA(I,J,KPO),THETA(IPO,J,KPO),
    4 THETA(I,JPO,KPO),THETA(IPO,JPO,KPO),THETA,IOP,T)
    CONTINUE
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 ROLL(I,J,K),ROLL(IPO,J,K),ROLL(I,J,KPO),
    3 ROLL(I,JPO,K),ROLL(I,J,KPO),ROLL(IPO,J,KPO),
    4 ROLL(I,JPO,KPO),ROLL(IPO,JPO,KPO),P,IOP,T)
    CONTINUE
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))

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C
2 PITCH(I,J,K),PITCH(IPO,J,K),PITCH(I,J,KPO),
3 PITCH(IPO,JPO,K),PITCH(I,J,KPO),PITCH(IPO,J,KPO),
4 PITCH(I,JPO,KPO),PITCH(IPO,JPO,KPO),Q,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 YAWR(I,J,K),YAWR(IPO,J,K),YAWR(I,J,KPO),
    3 YAWR(IPO,JPO,K),YAWR(I,J,KPO),YAWR(IPO,J,KPO),
    4 YAWR(I,JPO,KPO),YAWR(IPO,JPO,KPO),R,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 THROTT(I,J,K),THROTT(IPO,J,K),THROTT(I,J,KPO),
    3 THROTT(IPO,JPO,K),THROTT(I,J,KPO),THROTT(IPO,J,KPO),
    4 THROTT(I,JPO,KPO),THROTT(IPO,JPO,KPO),THRO,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 ELEVAT(I,J,K),ELEVAT(IPO,J,K),ELEVAT(I,J,KPO),
    3 ELEVAT(IPO,JPO,K),ELEVAT(I,J,KPO),ELEV(IPO,J,KPO),
    4 ELEVAT(I,JPO,KPO),ELEVAT(IPO,JPO,KPO),ELV,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 AILERON(I,J,K),AILERON(IPO,J,K),AILERON(I,J,KPO),
    3 AILERON(IPO,JPO,K),AILERON(I,J,KPO),AILERON(IPO,J,KPO),
    4 AILERON(I,JPO,KPO),AILERON(IPO,JPO,KPO),AIL,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 RUDDER(I,J,K),RUDDER(IPO,J,K),RUDDER(I,J,KPO),
    3 RUDDER(IPO,JPO,K),RUDDER(I,J,KPO),RUDDER(IPO,J,KPO),
    4 RUDDER(I,JPO,KPO),RUDDER(IPO,JPO,KPO),RUD,IOP,T)
C
    CALL LINT3D(H,XM,XLFAC,XDAT(1),XDAT(2),YDAT(1),
    1 YDAT(2),ZDAT(1),ZDAT(2))
    2 DTAIL(I,J,K),DTAIL(IPO,J,K),DTAIL(I,J,KPO),
    3 DTAIL(IPO,JPO,K),DTAIL(I,J,KPO),DTAIL(IPO,J,KPO),
    4 DTAIL(I,JPO,KPO),DTAIL(IPO,JPO,KPO),DTL,IOP,T)
C
    COORDINATED TURN ASSUMED
    BET=0.D0
    IERR=0
    GO TO 20000
C
    10000 IERR=1
    20000 RETURN
    END
C
3-D LINEAR INTERPOLATION : IF IOPT=0
NO INTERPOLATION ALONG THE Z DIRECTION
DATA IS ASSUMED TO BE GIVEN AT THE EIGHT
CORNERS OF THE CUBE WITHIN WHICH THE ACTUAL
POINT IS LOCATED
SUBROUTINE LINT3D(X,Y,Z,XR1,XR2,YR1,YR2,ZR1,ZR2,
1 A111,A211,A212,A112,A212,A122,A222,AOUT,IOP,T)
IMPLICIT REAL*8(A-H,O-Z)
C

```

```

C C 3-D INTERPOLATION : RIGHT HANDED TRIAD
C C X-ALTITUDE
C C Y-MACH NO.
C C Z-LOAD FACTOR
C
C DX=XR2-XR1
C DY=YR2-YR1
C DZ=ZR2-ZR1
C
C XD=X-XR1
C YD=Y-YR1
C ZD=Z-ZR1
C
C -----
C INTERPOLATE AT THE FIRST LOAD FACTOR
C INTERPOLATE ALONG ALTITUDE AT THE FIRST
C MACH NO. : A111,A211
C
C V11=A111*((A211-A111)/DX)*XD
C
C INTERPOLATE ALONG ALTITUDE AT THE SECOND
C MACH NO. : A121,A221
C
C V21=A121*((A221-A121)/DX)*XD
C
C INTERPOLATE ALONG THE MACH DIRECTION
C
C V1=V11+((V21-V11)/DY)*YD
C IF (IGET.EQ.0) AOUT=V1
C IF (IGET.EQ.0) GO TO 1000
C
C INTERPOLATE AT THE SECOND LOAD FACTOR
C -----
C INTERPOLATE ALONG ALTITUDE AT THE FIRST
C MACH NO. : A112,A212
C
C V12=A112*((A212-A112)/DX)*XD
C
C INTERPOLATE ALONG THE ALTITUDE AT THE SECOND
C MACH NO. : A122,A222
C
C V22=A122*((A222-A122)/DX)*XD
C
C INTERPOLATE ALONG THE MACH DIRECTION
C
C V2=V12+((V22-V12)/DY)*YD
C
C INTERPOLATE ALONG THE LOAD FACTOR DIRECTION
C -----
C IF (DZ.LE.0.D0) AOUT=V1
C IF (DZ.LE.0.D0) GO TO 1000
C AOUT=V1+((V2-V1)/DZ)*ZD
C
C 1000 RETURN
C END
C
C ATMOSPHERE MODEL FROM 10000' TO 50000'
C
C SUBROUTINE ATMO(ALTITU,V SOUND, DENSITY, VISCO, DVSDH, DDOH, DVDH)
C
C IMPLICIT REAL*8(A-H,O-Z)
C DIMENSION ALT(9), SPEED(9), DENSIT(9), VIS(9)

```

```

C C DATA I/I/
C C REFERENCE ALTITUDES
C DATA ALT/10000.D0,15000.D0,20000.D0,25000.D0,30000.D0,
C 1 35000.D0,40000.D0,45000.D0,50000.D0/
C
C SPEED OF SOUND
C DATA SPEED/1077.40076,1057.33533,1036.92444,1016.08020,
C 1 994.84094,972.93378,968.07001,968.07001,
C 2 968.07001/
C
C AIR DENSITY
C DATA DENSITY/0.00175583,0.00149777,0.00126774,0.00106755,
C 1 0.00089129,0.00073921,0.00058837,0.00046347,
C 2 0.00036471/
C
C VISCOSITY
C DATA VIS/1.137,1.104,1.070,1.035,1.000,0.9635,0.9555,
C 1 0.9555,0.9555/
C
C 10 IF (ALTITU.GT.ALT(I+1)) I=I+1
C 20 IF (ALTITU.LT.ALT(I)) I=I-1
C IF (ALTITU.LT.ALT(I)) GO TO 20
C K=I+1
C DH=ALT(K)-ALT(I)
C RH=ALTITU-ALT(I)
C DSPEED=(SPEED(K)-SPEED(I))/DH
C VSOUND=SPEED(I)+DSPEED*RH
C DDEN=(DENSITY(K)-DENSITY(I))/DH
C DENSITY=DENSITY(I)+DDEN*RH
C DVIS=(VIS(K)-VIS(I))/DH
C VISCO=VIS(I)+DVIS*RH
C VISCO=VISCO/32.14352D0
C
C PARTIAL DERIVATIVE OF SONIC SPEED, DENSITY AND
C VISCOSITY WITH RESPECT TO ALTITUDE
C
C DVSDH=DSPEED
C DDOH=DDEN
C DVDH=DVIS/32.14352D0
C
C RETURN
C END

```

APPENDIX D

LINEAR TIME VARYING SIMULATION  
IN SYSTEM\_BUILD

## LINEAR TIME VARYING SIMULATION IN SYSTEM BUILD

Linear time varying simulation can be built around a Fortran block evaluating the matrix equation

$$y = A(t)X + B(t)U \quad (D.1)$$

where  $y \in \mathbb{R}^l$ ,  $X \in \mathbb{R}^n$ ,  $U \in \mathbb{R}^m$ . A and B are time varying matrices of compatible dimensions. To use this Fortran block, the A and B matrix elements must be arranged in the following tabular form and transferred to Matrix<sub>x</sub> stack.

$$\text{Table} = \begin{bmatrix} t_0 & A_{11} & A_{21} & \dots & A_{n1} & A_{12} & A_{22} & \dots & A_{n2} & \dots & A_{nn} & B_{11} & B_{21} & B_{n1} & \dots & B_{nm} \\ t_1 & A_{11} & A_{21} & \dots & A_{n1} & A_{12} & A_{22} & \dots & A_{n2} & \dots & A_{nn} & B_{11} & B_{21} & B_{n1} & \dots & B_{nm} \\ t_2 & A_{11} & A_{21} & \dots & A_{n1} & A_{12} & A_{22} & \dots & A_{n2} & \dots & A_{nn} & B_{11} & B_{21} & B_{n1} & \dots & B_{nm} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \end{bmatrix} \quad (D.2)$$

↑  
independent  
variable

To build in the Fortran block, two distinct steps are necessary. The first one consists of building a dummy superblock with the name TABLE(N), where N could be any number from 0 to 9. The Table D.2 is built as a linear interpolation table block in this superblock, say at relative location M.

Next, the superblock(s) including the time varying Fortran block may be built with the following responses for the queries

Dimension of RP & IP: 0,2 (no real parameters, two real parameters)  
enter IP : N,M (Dummy superbloc Table number, relative  
location)

Note that to complete the time varying simulation, one has to define an independent string of  $l$  integrators. A schematic flow chart for building the time varying linear system with this FORTRAN block is given in Figure D.1.

The SYSTEM\_BUILD<sup>TM</sup> block diagram for linear time varying simulation for the aircraft example discussed in this Appendix is given in Figure D.1. The first block in this diagram computes the derivatives of the states using the Fortran block. These are then integrated in block 2. Block 3 essentially routes various inputs and outputs. The reference trajectory is generated in Block 4 while the wind is generated in Block 5. Block 6 implements the time varying gains or gain schedules, again using the Fortran block.



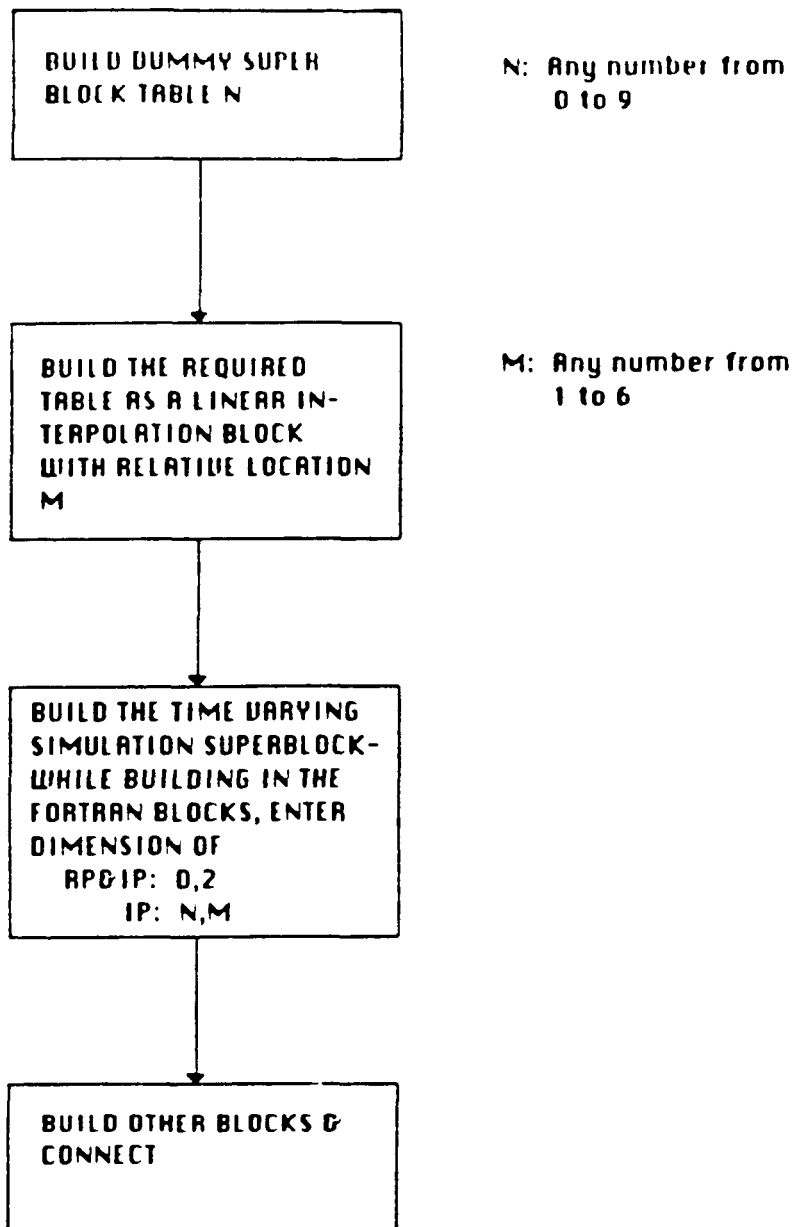


Figure D.1. Flow Chart for Building Linear Time Varying Simulation on  
SYSTEM\_BUILD

## APPENDIX E

### MINIMUM ERROR EXCITATION OUTPUT FEEDBACK DESIGNS

## MINIMUM ERROR EXCITATION OUTPUT FEEDBACK DESIGNS

This appendix gives the performance index specification for the control laws designed at each flight condition. Together with the algorithm description of section 4.2.2, these performance indices completely specify how the 30 sets of output gains can be reproduced.

### LQR Performance Index

$$J = \int_0^{\infty} e^{-2\alpha t} (x^T H A_y^T H x + u^T R_{uu} u) dt, \text{ with}$$

$$A_y = \text{Diag} \left[ \frac{1}{(50)^2}, \frac{1}{(0.01)^2}, \frac{1}{(0.001)^2}, \frac{1}{(0.001)^2}, \frac{1}{(0.001)^2}, \frac{1}{(0.001)^2}, \right. \\ \left. \frac{1}{(0.001)^2}, \frac{1}{(0.001)^2}, \frac{1}{(0.001)^2}, \frac{1}{(0.005)^2}, \frac{1}{(0.005)^2}, \right. \\ \left. \frac{1}{(0.005)^2} \right]$$

State weighting matrix,  $R_{xx} = k H^T A_y^T H$ ,

here:  $k$  is a scalar providing relative state-control weights

$H$  is the system output matrix

The control weighting matrix  $R_{uu} = \text{diag}^2(B)$

Table D-1 lists the scalar weight  $k$  and the state weighting vector  $B$  as a function of the design flight condition.

TABLE E-1. PERFORMANCE INDEX SPECIFICATION FOR EACH FLIGHT CONDITION  
(Subscript 1 corresponds to the design with all controls,  
subscript 2 corresponds to the design without throttle  
control.)

LOAD FACTOR	ALTITUDE AND MACH				
	h = 10k M = 0.8	h = 20k M = 1.0	h = 30k M = 1.2	h = 40k M = 1.4	h = 50k M = 1.8
1	$B_1 = [0.1 \ 1 \ 1 \ 1]$ $k_1 = 0.002$	$B_1 = [0.1 \ 1 \ 1 \ 1]$ $k_1 = 0.002$	$B_1 = [0.1 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [0.1 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [0.1 \ 1 \ 1 \ 1]$ $k_1 = 0.001$
	$B_2 = [1 \ 1 \ 1]$ $k_2 = 0.002$	$B_2 = [1 \ 1 \ 1]$ $k_2 = 0.002$	$B_2 = [1 \ 1 \ 1]$ $k_2 = 0.005$	$B_2 = [1 \ 1 \ 1]$ $k_2 = 0.01$	$B_2 = [1 \ 1 \ 1]$ $k_2 = 0.01$
2	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.002$
	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.0005$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.001$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.001$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.01$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.00001$
4	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.001$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.0005$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.0005$	$B_1 = [5 \ 1 \ 1 \ 1]$ $k_1 = 0.00012$
	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.002$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.002$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.002$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.002$	$B_2 = [0.1 \ 0.1 \ 1]$ $k_2 = 0.00001$

## Output Feedback Gains

The output feedback Controller has the form

$$u = Cy$$

where

$$u = [\delta T \ \delta e_{ap} \ \delta a_{ap} \ \delta r_{ap}]^T$$

$$y = [\delta h \ \delta M \ \delta \alpha \ \delta \gamma \ \delta \phi \ \delta \beta \ \delta p \ \delta q \ \delta r \ \int \delta \alpha \ \int \delta \phi \ \int \int \delta \phi]^T$$

Flight condition:  $h=10k$  ft,  $M=0.8$ ,  $n=1$

(a) Throttle free

COLUMNS 1 THRU 6					
6.1491D-04	1.9996D+03	1.2553D+01	3.9128D+03	3.0025D-02	7.0112D-02
-2.1757D-02	9.3094D+00	-4.8115D+01	-3.8109D+01	-5.8576D-01	-3.0981D-01
-8.5700D-05	2.7476D-02	-2.5024D-01	-1.2745D-01	5.8288D+01	-9.7112D+01
-3.4216D-06	1.3714D-03	-9.5302D-03	-5.2506D-03	6.1841D-01	3.3190D+00

COLUMNS 7 THRU 12					
-3.5007D-02	2.7833D+00	-6.4362D-03	-5.8824D+03	1.8056D-02	3.6150D-03
-3.7222D-01	-2.5667D+01	1.6186D-01	-6.4055D+01	-3.1782D-01	-5.4163D-02
2.3275D+01	-1.5444D-01	2.3048D+01	-2.4475D-01	3.2259D+01	5.4872D+00
8.1391D-01	-7.5431D-03	-4.3978D+00	-1.0375D-02	3.7226D-01	6.3560D-02

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-6.9831D-01	-1.7886D+04	-9.4324D+01	-2.5680D+02	-1.9276D-01	-2.7120D-01
-1.9239D-03	-4.9332D+01	-2.6985D-01	-6.9729D-01	5.8289D+01	-9.7113D+01
-1.5775D-05	-4.0474D-01	-2.2771D-03	-5.6596D-03	6.1835D-01	3.3189D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.5857D-01	-2.0762D+01	2.0846D-01	0.0000D+00	-9.4988D-02	-1.5752D-02
2.3276D+01	-1.2968D-01	2.3048D+01	0.0000D+00	3.2260D+01	5.4873D+00
8.1388D-01	-9.4264D-04	-4.3978D+00	0.0000D+00	3.7223D-01	6.3555D-02

Flight Condition: h=10k ft, M=0.8, n=2

(a) Throttle free

COLUMNS 1 THRU 6					
2.9627D-02	8.1168D-02	-2.8592D-02	-7.8192D-01	-1.3855D-01	-3.8065D-01
-8.2939D-02	-1.7109D+03	-4.3457D+01	-4.2727D+01	-3.0855D+00	-5.8524D-01
-6.9160D-03	-3.0315D+02	5.3834D+00	3.8716D+01	4.1267D+01	-4.6182D+01
-3.5156D-04	-6.7290D+01	3.0272D+00	1.6798D+01	-9.9049D-01	1.2926D+01

COLUMNS 7 THRU 12					
-8.1273D-03	-3.1193D-01	6.5570D-02	7.7035D+01	-2.8132D-01	-8.7290D-02
-1.6231D+00	-1.0912D+01	-2.1829D+00	-1.9868D+02	-6.1947D-01	-5.4515D-01
1.7513D+01	1.8824D+00	-3.1690D+00	-4.6712D+01	2.4901D+01	4.0680D+00
-1.5747D-01	7.6861D-02	-3.2354D+00	-1.4266D+01	7.7021D-01	-7.0858D-03

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-9.4567D-01	-2.2554D+04	-1.7541D+02	-3.7794D+02	8.7822D+01	-1.3809D+02
-5.4520D-02	-1.5096D+03	3.5769D+00	2.0487D+01	1.7270D+02	-6.2175D+01
-1.8692D-02	-5.2788D+02	2.5422D+00	5.5214D+00	6.6557D+00	5.6499D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
4.0991D+01	-1.2821D+02	9.6703D+01	-2.1918D+03	3.4846D+01	6.0921D+00
8.3971D+01	1.7401D+00	-1.9941D+00	-1.5238D+02	9.8190D+01	1.6852D+01
3.9233D+00	-3.6830D+00	2.7026D+00	-5.4415D+01	4.5116D+00	7.3470D-01

Flight Condition: h=10k ft, M=0.8, n=4

(a) Throttle free

COLUMNS 1 THRU 6					
2.2794D-03	7.1210D+01	4.0241D-01	-1.1510D+00	-6.1276D-02	-2.4384D-01
-4.3918D-02	-9.4806D+02	-4.5341D+01	-8.3475D+00	-1.6230D+01	1.4902D+01
-3.3214D-04	-2.5846D+02	2.8570D+00	5.7110D+01	3.9784D+01	-4.6713D+01
2.0198D-03	-6.1265D+01	1.0238D+00	2.2273D+01	-2.9065D+00	1.5259D+01

COLUMNS 7 THRU 12					
9.7857D-03	-8.7439D-02	1.9166D-02	1.8054D+01	-3.3040D-01	-1.2460D-01
-1.7379D-01	-7.5154D+00	-1.8527D+00	-2.8085D+02	7.6036D+00	6.2348D-02
1.6825D+01	2.3431D+00	-3.2140D+00	-7.6799D+01	2.9976D+01	5.1782D+00
-6.3254D-01	2.4454D-01	-3.4999D+00	-2.0067D+01	2.1467D+00	2.4978D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.4868D-01	-4.5480D+03	-1.9039D+02	-1.8448D+02	5.2790D+01	-3.5337D-01
-1.3103D-02	-2.0613D-02	1.2893D+00	1.7878D+01	2.6314D+02	-6.6656D+01
1.9612D-03	-3.6559D+01	1.6883D-01	1.4573D+01	2.4480D+00	8.1336D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
2.7615D+01	-1.6666D+02	4.0532D+01	-1.3112D+03	2.1815D+01	3.1686D+00
1.3028D+02	4.3779D+00	-2.2701D+00	-5.5957D+01	1.5131D+02	2.6108D+01
8.0487D-01	-6.8981D-01	-2.3050D+00	-1.2346D-01	3.0843D+00	4.9332D-01

Flight Condition: h=20k ft, M=1.0, n=1

(a) Throttle free

COLUMNS 1 THRU 6					
-3.0028D-02	1.4232D+03	-1.4150D+02	1.2419D+04	1.5656D-01	2.5076D-01
-2.4707D-02	3.9892D+00	-1.3192D+02	2.1845D+01	-2.2288D-01	-3.9225D-01
-6.2268D-05	1.0190D-02	-2.5252D-01	6.6599D-02	5.6690D+01	-1.0369D+02
-1.5270D-06	3.3189D-04	-6.0395D-03	1.8372D-03	1.0355D+00	3.7093D+00

COLUMNS 7 THRU 12					
-3.1757D-02	-9.4699D+00	-2.8527D-02	-1.7748D+04	8.6553D-02	1.6153D-02
-1.9513D-01	-2.3818D+01	2.9430D-01	-1.8270D+02	-1.2200D-01	-2.0972D-02
2.2984D+01	-1.3557D-01	2.5598D+01	-4.6193D-01	3.1368D+01	5.3333D+00
1.0205D+00	-4.0981D-03	-4.4825D+00	-1.2099D-02	6.0750D-01	1.0334D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.4623D-01	-3.0862D+03	-1.3772D+02	-2.5546D+02	-2.3182D-01	-3.5661D-01
-3.4606D-04	-7.3324D+00	-3.2365D-01	-5.9666D-01	5.6691D+01	-1.0369D+02
-8.5753D-06	-1.8118D-01	-7.6613D-03	-1.4886D-02	1.0355D+00	3.7093D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.8998D-01	-2.1146D+01	2.7138D-01	0.0000D+00	-1.1843D-01	-2.0058D-02
2.2985D+01	-1.4990D-01	2.5599D+01	0.0000D+00	3.1368D+01	5.3334D+00
1.0205D+00	-4.2346D-03	-4.4824D+00	0.0000D+00	6.0749D-01	1.0334D-01

Flight Condition: h=20k ft, M=1.0, n=2

(a) Throttle free

COLUMNS 1 THRU 6					
6.9042D-03	1.9549D+02	1.5364D+00	-8.5917D-01	1.1123D-01	-4.6072D-01
-8.0054D-02	-1.6813D+03	-9.5674D+01	-3.7453D+01	-1.7710D+01	1.1887D+01
-3.1811D-03	-1.9103D+02	7.1812D+00	4.0158D+01	4.1578D+01	-4.8085D-01
7.3580D-04	-2.9358D+01	3.8067D+00	1.8625D+01	-1.0029D+00	1.2088D+01

COLUMNS 7 THRU 12					
1.1787D-01	-3.2178D-02	1.3241D-01	1.8588D+01	-1.5842D-01	-5.7223D-02
-1.2003D+01	-1.7947D+01	-8.0714D+00	-2.1281D+02	-7.5076D+00	-1.4183D+00
1.7896D+01	1.8398D+00	-2.7051D+00	-3.4723D+01	2.4838D+01	4.0935D+00
8.7576D-03	-2.7686D-02	-3.0579D+00	-1.0897D+01	9.1879D-01	-1.1976D-02

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.5821D-01	-5.1355D+03	-1.8856D+02	-2.4846D+02	2.3176D+01	-5.6307D+01
-3.3381D-03	-1.8932D+02	8.8631D+00	3.0696D+01	2.0976D+02	-5.8322D+01
-1.4984D-03	-9.3411D+01	1.1209D+00	1.3272D+01	2.8245D+00	7.2034D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
9.7017D+00	-1.1540D+02	2.1620D+01	-5.6373D+02	6.8088D+00	1.0118D+00
1.0345D+02	7.8005D+00	-3.3573D+00	-2.3474D+01	1.1937D+02	2.0505D+01
1.2950D+00	-1.1666D+00	-2.2116D+00	-1.4181D+01	2.4988D+00	3.6095D-01

Flight Condition: h = 20k ft, M=1.0, n=4

(a) Throttle free

COLUMNS 1 THRU 6					
-2.9947D-05	3.6599D+00	5.5165D-01	-4.3026D-01	1.0464D-01	-1.4545D-01
-2.4350D-02	-1.6104D+02	-9.3784D+01	-5.3770D+01	9.3674D+00	-2.0784D+01
8.9437D-03	-5.3990D+01	5.8315D+00	6.7524D+01	3.8510D+01	-4.8181D+01
4.1491D-03	-1.8467D+01	5.9942D-01	2.9591D+01	-3.6880D+00	1.6136D+01

COLUMNS 7 THRU 12					
1.7035D-02	1.0704D-01	1.9226D-02	1.4165D+00	-3.8941D-02	-1.7625D-02
3.7699D-01	-2.8745D+01	1.1755D+00	-9.7631D+01	4.2783D-01	2.9496D-01
1.6541D+01	1.7075D+00	-3.4304D+00	-2.9706D+01	2.9120D+01	5.0757D+00
-1.9703D+00	-3.2254D-01	-4.4090D+00	-1.1588D+01	9.5902D-01	1.5582D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-6.0133D-02	-4.3196D+02	-1.5452D+02	-1.3945D+02	1.1041D+01	-4.5528D+01
1.3187D-03	9.5875D+00	1.5789D+01	3.1355D+01	2.5958D+02	-6.5193D+01
3.5796D-03	-4.9691D-01	1.9333D+00	1.9762D+01	3.1445D-01	7.3005D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
3.4082D+00	-1.4477D+02	3.1317D+00	-2.7722D+02	-5.3801D+00	-1.2555D+00
1.2898D+02	1.7756D+01	-3.4610D+00	9.5732D+00	1.4989D+02	2.5871D+01
-4.7916D-01	2.1208D+00	-4.1433D+00	-1.0686D+00	1.6873D+00	2.9084D-01

Flight condition: h = 30k ft, M=1.2, n=1

(a) Throttle free

COLUMNS 1 THRU 6					
-4.6088D-02	1.6086D+03	-4.0862D+01	7.7420D+03	5.0626D-01	2.2822D-01
-5.7504D-02	-2.4481D+01	-1.9418D+02	5.1413D+01	1.9622D+00	4.4702D-01
-4.0537D-05	1.7568D-02	-2.0662D-01	1.4692D-01	4.2962D+01	-7.5834D+01
-7.3106D-08	1.2564D-04	-2.9104D-04	2.9682D-04	-2.2994D-01	8.0850D+00

COLUMNS 7 THRU 12					
2.5531D-01	1.9677D+01	-8.0973D-03	-6.7504D+03	2.9316D-01	5.1002D-02
1.0298D+00	-5.9481D+00	1.6625D-01	-2.3518D+02	1.1180D+00	1.9185D-01
1.6729D+01	-7.8761D-02	1.1728D+01	-3.0407D-01	2.3583D+01	4.0053D+00
6.7563D-01	-1.0035D-03	-5.5422D+00	-7.5542D-04	-4.1457D-02	-4.9060D-03

(b) Fixed Throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.2035D-01	-4.5955D+03	-2.4374D+02	-4.8065D+02	1.4541D+00	-1.1746D-01
-3.9297D-04	-8.2035D+00	-3.9043D-01	-8.4043D-01	7.7573D+01	-9.1042D+01
-9.4818D-06	-1.9769D-01	-9.3204D-03	-2.0343D-02	1.9012D+00	2.6540D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
7.5137D-01	-1.7241D+01	2.9844D-01	0.0000D+00	8.2581D-01	1.4191D-01
3.4197D+01	-1.5816D-01	2.8032D+01	0.0000D+00	4.3223D+01	7.3793D+00
1.3758D+00	-4.1546D-03	-5.1448D+00	0.0000D+00	1.0546D+00	1.7917D-01



Flight Condition: h=30k ft, M=1.2, n=2

(a) Throttle free

COLUMNS 1 THRU 6					
6.6424D-03	2.0838D+02	2.4550D+00	-4.6783D+00	2.7707D+00	-2.5482D+00
-7.9027D-02	-1.9271D+03	-1.7763D+02	3.9364D+01	-1.1536D+02	5.3945D+01
-2.5690D-03	-2.0704D+02	9.0405D+00	4.7758D+01	4.4819D+01	-3.8343D+01
1.9215D-03	-2.3017D+00	6.6459D+00	1.9007D+01	2.1634D+00	7.7438D+00

COLUMNS 7 THRU 12					
1.3539D+00	1.8265D-02	1.0172D+00	4.1165D+01	9.1485D-01	1.2343D-01
-8.0765D+01	-1.9153D+01	-3.1558D+01	-4.4553D+02	-5.0057D+01	-7.8047D+00
1.8876D+01	1.3449D+00	-3.4281D+00	-5.0063D+01	2.6362D+01	4.3945D+00
2.4501D+00	5.2374D-01	-1.5152D+00	-3.9816D+00	3.1590D+00	2.5665D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-2.7603D-01	-5.6529D+03	-2.9060D+02	-3.6612D+02	5.2588D+01	-8.8793D+01
-1.0980D-02	-4.1304D+02	3.7056D+00	4.2548D+01	2.1125D+02	-3.8144D+01
-2.3001D-03	-1.2616D+02	1.4133D+00	1.8791D+01	3.0150D+00	9.2470D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.8824D+01	-9.4044D+01	4.0305D+01	-1.1392D+03	2.0379D+01	3.5850D+00
1.0450D+02	4.1414D+00	-7.5652D+00	-8.7991D+01	1.2094D+02	2.0762D+01
2.2519D+00	-2.2669D+00	-2.0442D+00	-2.9238D+01	3.5012D+00	4.7414D-01

Flight Conditions: h=30k ft, M=1.2, n=4

(a) Throttle free

COLUMNS 1 THRU 6					
4.1769D-04	3.6498D+01	1.4851D+00	-3.4841D+00	2.1042D+00	-1.3394D+00
-6.6101D-02	-1.0172D+03	-2.1118D+02	-1.4182D+02	-1.1462D+01	-7.4184D+01
6.4192D-03	-2.8674D+02	4.5654D+00	1.1561D+02	2.7896D+01	-2.6873D+01
4.5633D-03	-4.2698D+01	4.9646D+00	4.3199D+01	-6.6763D+00	2.0181D+01

COLUMNS 7 THRU 12					
6.3631D-01	4.8008D-02	3.3416D-01	1.4069D+01	5.3312D-01	9.1566D-02
-3.1037D+01	-2.6264D+01	1.7762D+00	-4.0761D+02	3.2469D+00	7.1053D+00
1.0313D+01	8.3152D-01	-4.9400D+00	-1.1891D+02	2.7199D+01	4.8492D+00
-2.4702D+00	3.2870D-01	-4.1412D+00	-2.0022D+01	9.2925D-01	-2.3495D-02

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.4369D-01	-2.6046D+03	-2.7650D+02	-2.2663D+02	1.8778D+01	-6.0432D+01
-1.8925D-03	-1.9404D+02	2.7848D+00	6.7997D+01	2.8567D+02	-3.9003D+01
2.3785D-03	-4.3929D+01	-1.3042D+00	2.6549D+01	-4.3513D+00	1.0744D+01

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
3.3267D+00	-1.3238D+02	2.5597D+01	-1.0794D+03	-2.4847D+00	-7.8123D-01
1.4358D+02	5.4095D+00	-1.5841D+01	-7.9803D+01	1.6880D+02	2.9097D+01
-2.1208D+00	-8.7326D-01	-4.3539D+00	-1.9425D+01	2.8894D-01	4.0536D-01

Flight Condition: h=40k ft, m=1.4, n=1

(a) Throttle free

COLUMNS 1 THRU 6					
-4.0290D-02	1.4738D+03	-7.8132D+01	1.2733D+04	2.5962D+00	4.7039D-01
-3.3645D-02	-3.4998D+01	-2.2104D+02	2.6056D+02	8.6121D+00	1.6425D+00
-2.2849D-05	1.0441D-02	-1.9676D-01	4.0965D-01	4.4062D+01	-5.9481D+01
7.4382D-07	-2.6323D-04	6.3096D-03	-1.3187D-02	-1.5759D+00	1.1276D+01

COLUMNS 7 THRU 12					
1.5059D+00	2.1803D+01	1.5995D-01	-7.5485D+03	1.4565D+00	2.4843D-01
4.9056D+00	-1.1218D+00	4.6368D-01	-3.3457D+02	4.7969D+00	8.1530D-01
1.8275D+01	-7.3752D-02	6.8008D+00	-3.9080D-01	2.4078D+01	4.0875D+00
1.2100D-01	1.6192D-03	-7.5773D+00	1.2509D-02	-7.5866D-01	-1.2434D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.7781D-01	-1.0549D+04	-1.9624D+02	-9.2410D+02	1.1114D+01	6.7530D-01
-3.6355D-04	-2.1593D+01	-3.6862D-01	-1.8757D+00	1.0197D+02	-7.4840D+01
-5.8903D-06	-3.4989D-01	-5.9813D-03	-3.0397D-02	2.2552D+00	6.3920D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
6.0610D+00	-1.7085D-01	9.1804D-02	0.0000D+00	6.2255D+00	1.0664D+00
4.7744D+01	-1.6007D-01	2.1718D+01	0.0000D+00	5.6830D+01	9.7189D+00
1.7109D+00	-2.6749D-03	-8.6307D+00	0.0000D+00	1.2089D+00	2.0712D-01

Flight Condition: h=40k ft, m=1.4, n=2

(a) Throttle free

COLUMNS 1 THRU 6					
6.8518D-03	5.0683D+02	4.0818D+00	-5.6239D+00	3.0230D+00	-1.7841D+00
-4.4551D-02	-2.3251D+03	-1.8765D+02	-7.0742D+01	-5.8678D+01	1.4762D+00
-9.5491D-04	-3.2593D+02	9.9991D+00	7.9025D+01	4.3050D+01	-3.0872D+01
2.1000D-03	-1.5805D+01	9.0181D+00	4.9655D+01	-4.8933D+00	1.5230D+01

COLUMNS 7 THRU 12					
1.4281D+00	-1.3268D-01	7.7201D-01	8.0912D+01	7.3992D-01	7.2793D-02
-6.5057D+01	-1.5583D+01	-1.2903D+01	-4.1372D+02	-2.4749D+01	-1.9782D+00
1.8056D+01	1.2552D+00	-4.8808D+00	-6.2475D+01	2.5905D+01	4.2967D+00
-6.8358D-01	5.6624D-01	-5.3267D+00	-8.8420D+00	6.9321D-01	-3.1452D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-4.5737D-01	-2.4801D+04	-7.2826D+02	-1.2147D+03	4.2798D+02	-2.8449D+02
-2.7317D-02	-2.0654D+03	-4.2632D+00	8.0166D+01	4.9573D+02	-2.0063D+01
-1.3001D-02	-8.8348D+02	-1.3085D+01	1.1110D-01	1.9376D+01	7.4667D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
2.0012D+02	-3.0735D+02	1.5434D+02	-4.0026D+03	1.9950D+02	3.6184D+01
2.5032D+02	7.0155D+00	-8.5866D+00	-3.3909D+02	2.8377D+02	4.8903D+01
1.2102D+01	-1.0560D+01	-6.1567D-01	-1.4738D+02	1.4019D+01	2.1601D+00

Flight Condition: h=40k ft, m=1.4, n=4

(a) Throttle free

COLUMNS 1 THRU 6					
4.1124D-04	5.0036D+01	1.5631D+00	-3.1841D+00	1.3668D+00	-5.9777D-01
-2.3227D-02	-1.1466D+03	-1.6878D+02	-5.7583D+01	-8.9560D+00	-1.7275D+01
4.3573D-03	-3.1217D+02	1.0447D+00	1.4759D+02	2.6414D+01	-2.7939D+01
3.9995D-03	-4.5419D+01	4.3753D+00	7.6267D+01	-1.0108D+01	2.5654D+01

COLUMNS 7 THRU 12					
3.5106D-01	-4.7050D-02	2.0038D-01	1.5471D+01	5.5993D-02	1.5265D-02
-3.3450D+01	-1.5111D+01	-8.9972D+00	-3.6314D+02	1.6913D+01	1.3191D+01
1.0563D+01	-6.9894D-01	-4.0463D+00	-1.0337D+02	2.8584D+01	5.1314D+00
-3.8184D+00	2.8959D-02	-6.4589D+00	-1.7890D+01	1.5474D+00	2.1119D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-7.4833D-02	-3.1628D+03	-3.0725D+02	-2.2405D+02	5.2112D+01	-4.3588D+01
-3.1778D-03	-2.5123D+02	-1.0373D+00	5.6016D+01	2.9213D+02	-4.4807D+01
2.4617D-03	-5.0406D+01	-7.3654D-01	5.1439D+01	-1.3608D+01	1.6665D+01

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.7193D+01	-1.2755D+02	9.9949D+00	-1.0546D+03	1.3909D+01	2.5306D+00
1.4775D+02	6.0470D+00	-6.7478D+00	-8.0806D+01	1.7152D+02	2.9610D+01
-6.2548D+00	-3.7364D-01	-8.3168D+00	-1.8416D+01	-2.9529D+00	-5.2277D-01

Flight Condition: h=50k ft, m=1.8, n=1

(a) Throttle free

COLUMNS 1 THRU 6					
-1.2428D-02	1.4946D+03	-1.7242D+02	1.8657D+04	6.6881D-01	2.9641D-01
-2.5982D-02	-1.2215D+01	-1.7927D+02	-1.7245D+01	8.9805D-01	1.0008D-01
-1.5234D-05	4.5541D-03	-1.6558D-01	6.9925D-02	4.4274D+01	-4.7749D+01
2.6681D-07	1.5175D-05	2.4938D-03	-1.1575D-03	-2.1426D+00	1.2960D+01

COLUMNS 7 THRU 12					
3.1504D-01	-1.1959D+01	-7.9929D-02	-8.0056D+03	3.8123D-01	6.6208D-02
4.4714D-01	-1.2251D+01	8.1814D-02	-1.2989D+02	5.1817D-01	8.9977D-02
1.9214D+01	-6.8022D-02	4.8313D+00	-1.4003D-01	2.4148D+01	4.0977D+00
-1.8412D-01	-3.1292D-04	-9.0948D+00	2.1152D-03	-1.0517D+00	-1.7169D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-3.9688D-01	-3.8011D+04	-1.7061D+02	-3.6535D+03	2.5598D+02	2.4961D+01
-8.5073D-04	-8.1553D+01	-3.4003D-01	-7.8179D+00	1.0221D+02	-5.7779D+01
-6.3987D-06	-6.1356D-01	-2.5685D-03	-5.8789D-02	1.4993D+00	1.0648D+01

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.4115D+02	-1.6571D+01	-9.4218D+00	0.0000D+00	1.4298D+02	2.4462D+01
4.8944D+01	-1.4756D-01	1.4088D+01	0.0000D+00	5.6839D+01	9.7199D+00
1.4580D+00	-1.1025D-03	-1.1752D+01	0.0000D+00	7.7799D-01	1.3455D-01

Flight Condition: h=50k ft, m=1.8, n=2

(a) Throttle free

COLUMNS 1 THRU 6					
3.5959D-03	4.6794D+02	3.5335D+00	-1.1105D+01	1.8649D+00	-8.0395D-01
-2.8089D-02	-1.8184D+03	-1.3708D+02	-1.4276D+02	3.3696D+01	-3.5020D+01
-2.7227D-03	-8.1445D+02	8.1206D+00	1.3217D+02	2.2062D+01	-2.0562D+01
-6.3217D-04	-4.5145D+02	7.5518D+00	1.0742D+02	-9.6043D+00	2.2742D+01

COLUMNS 7 THRU 12					
4.6028D-01	-3.5162D-01	2.6473D-01	7.7890D+01	1.0763D-01	-1.1116D-01
-1.6152D+01	-6.4189D+00	-2.1512D+00	-3.1546D+02	3.0005D+01	7.0649D+00
7.4751D+00	1.2381D+00	-1.7457D+00	-1.4860D+02	1.4924D+01	2.3107D+00
-4.0962D+00	6.7878D-01	-7.0306D+00	-8.6670D+01	-1.4925D+00	-7.2027D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-6.0982D-02	-4.1785D+03	-2.5111D+02	-2.8238D+02	-6.9991D+01	-7.1699D+01
-3.6150D-03	-5.4820D+02	-9.6721D-01	3.8862D+01	4.1818D+01	-4.1017D+01
-1.4051D-03	-2.8559D+02	4.3810D-01	3.2729D+01	-7.6396D+00	6.2683D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-7.2926D+01	-3.0622D+01	1.9139D+01	-7.2199D+02	-3.7145D+01	-5.9716D+01
1.8956D+01	6.3265D-01	2.6529D+00	-9.4700D+01	2.4460D+01	4.1671D+00
1.5019D+00	-9.5266D-02	-1.8004D+00	-5.0892D+01	-1.3886D-01	-1.6812D-01

Flight Condition: h=50k ft, m=1.8, n=4

(a) Throttle free

COLUMNS 1 THRU 6					
-8.7236D-06	3.3564D+01	1.3407D+00	-2.8980D+00	5.8841D-01	-2.1908D-01
-1.4706D-02	-9.3343D+02	-1.4934D+02	-3.1412D+01	3.4538D+01	-2.7494D+01
9.5740D-03	-2.8197D+02	-1.1045D+00	2.3251D+02	1.3507D+01	-2.1212D+01
6.8283D-03	-8.9545D+01	-2.3177D+00	1.4395D+02	-1.5042D+01	3.3745D+01

COLUMNS 7 THRU 12					
3.9985D-02	-6.0302D-02	5.0477D-02	1.1111D+01	-1.1816D-01	-8.0972D-03
-7.0423D+00	-1.6189D+00	-5.6683D+00	-3.0429D+02	5.0324D+01	1.7827D+01
5.1013D+00	7.2874D-01	-3.3023D-01	-1.0184D+02	2.4240D+01	4.4423D+00
-4.8778D+00	2.6307D-03	-7.9269D+00	-3.6055D+01	8.3040D-01	4.2546D-01

(b) Fixed throttle

COLUMNS 1 THRU 6					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.7721D-02	-9.5730D+02	-1.7563D+02	-9.2676D+01	5.9326D+01	-2.8669D+01
1.0931D-03	-8.2717D+01	-2.1945D+00	4.2145D+01	4.4064D+01	-6.5929D+01
1.2398D-03	-4.0076D+01	-3.3685D+00	3.5458D+01	-7.1816D+00	7.5126D+00

COLUMNS 7 THRU 12					
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
8.9326D+00	-1.3331D+01	9.2995D+00	-3.4701D+02	3.1186D+01	8.2989D+00
1.9945D+01	4.7141D-01	6.5710D+00	-2.9380D+01	2.7616D+01	4.7279D+00
-4.1092D-01	-4.0034D-01	-1.8719D+00	-1.5270D+01	-2.1025D+00	-1.6154D-01